



รายงานผลการวิจัย

การมีเสถียรภาพและค่าที่เหมาะสมที่สุดของแบบจำลองระบบประสาท

Stabilization and Optimization of Neural Networks

โดย

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Stabilization and Optimization of Neural Networks

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กิตติกรรมประกาศ

โครงการวิจัยเรื่อง การมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท (Stabilization and Optimization of Neural Networks) ได้สำเร็จลุล่วง โดยได้รับทุนอุดหนุนการวิจัยจากสำนักวิจัยและส่งเสริมวิชาการการเกษตร มหาวิทยาลัยแม่โจ้ ประจำปีงบประมาณ พ.ศ.2556 ผู้วิจัย ขอขอบคุณ Prof. Dr. Vu Ngoc Phat และ Prof. Dr. Norbert Herrmann ที่เป็นที่ปรึกษาในงานวิจัยครั้งนี้ ทำให้งานวิจัยมีความสมบูรณ์มากยิ่งขึ้น และขอขอบคุณคณะวิทยาศาสตร์ มหาวิทยาลัยแม่โจ้ ที่อนุเคราะห์เรื่องสถานที่ และอุปกรณ์บางอย่างที่ใช้ในการดำเนินการวิจัยให้เสร็จสิ้นสมบูรณ์ไปด้วยดี

เกรียงไกร ราชกิจ

กันยายน 2556



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Stabilization and Optimization of Neural Networks

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บทคัดย่อ

งานวิจัยนี้เป็นเรื่องที่เกี่ยวข้องกับปัญหาของการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท โดยการประยุกต์ใช้ฟังก์ชันไลปูนอฟในการแก้ปัญหามีเสถียรภาพและค่าเหมาะที่สุดและพัฒนาเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์ จากกรณีนี้เป็นเงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาทที่จะนำเสนอการปรับปรุงพัฒนาผลลัพธ์ของการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาทจะนำเสนอในตัวอย่าง

คำสำคัญ : แบบจำลองระบบประสาท เสถียรภาพ ค่าเหมาะที่สุด

Abstract

This research is concerned with the problem of Stabilization and Optimization of Neural Networks. The stability problem is solved by applying a Lypunov functional, and an improved delay-dependent stability criterion is obtained in terms of a linear matrix inequality. Based on this, a sufficient condition for stabilization of the system is presented. The reduced conservatism of the proposed stability result is shown through numerical examples.

Key words: Neural Networks, Stabilization, Optimization



คำนำ

เนื่องจากข้าพเจ้าได้ทำงานวิจัยและตีพิมพ์ผลงานวิจัยร่วมกับ Prof. Dr. Vu Ngoc Phat และ Prof. Dr. Dr. h.c. Norbert Herrmann ซึ่งเป็นผู้เชี่ยวชาญทางด้าน Control theory, Modeling and Optimization ที่มีผลงานวิจัยและตีพิมพ์มากมายเป็นที่ยอมรับในระดับนานาชาติและ ได้มีโอกาสเชิญท่านมาทำงานวิจัยร่วมที่มหาวิทยาลัยแม่โจ้ ระหว่างวันที่ 20 มิ.ย. 2553 จนถึง 17 ก.ค. 2553 และ 1-5 พฤษภาคม 2554 ซึ่งได้มีโอกาสแลกเปลี่ยนความคิดเห็นในการวิจัยและพัฒนางานวิจัยไปสู่ระดับนานาชาติร่วมกับคณาจารย์สาขาวิชาคณิตศาสตร์ มหาวิทยาลัยแม่โจ้ ดังนั้น ท่านได้แนะนำโครงการวิจัยเกี่ยวกับแบบจำลองโครงข่ายประสาทเทียม ซึ่งเป็นหัวข้อที่สนใจของนานาชาติและมีประโยชน์อย่างมากกับทางการแพทย์ อีกทั้งยังมีผู้ศึกษาทางด้านนี้เป็นจำนวนน้อยมากในประเทศไทย ดังนั้นข้าพเจ้าจึงได้สนใจที่จะทำงานวิจัยในเรื่อง Stabilization and Optimization for neural networks เพื่อสร้างองค์ความรู้ใหม่และพัฒนางานวิจัยทั้งระดับชาติและนานาชาติ อีกทั้งมีประโยชน์อย่างมากกับทางการแพทย์ และเผยแพร่สู่ชุมชนในระดับชาติและนานาชาติ และสร้างกลุ่มวิจัยทางด้าน Stabilization and Optimization for neural networks ร่วมกับคณาจารย์สาขาคณิตศาสตร์ มหาวิทยาลัยแม่โจ้ และความร่วมมือกับต่างประเทศ

โครงข่ายประสาทเทียมเป็นการจำลองการทำงานของสมองมนุษย์ ด้วยโปรแกรมคอมพิวเตอร์เป็นแนวความคิดที่ต้องการให้คอมพิวเตอร์มีความชาญฉลาดในการเรียนรู้เหมือนที่มนุษย์มีการเรียนรู้ สามารถฝึกฝนได้ และสามารถนำความรู้และทักษะไปแก้ปัญหาลักษณะต่าง ๆ มี นักวิจัยจำนวนมากได้คิดค้นรูปแบบโครงข่ายประสาทเทียมแบบต่าง ๆ เพื่อนำมาประยุกต์ใช้อย่าง กว้างขวาง การประยุกต์ใช้งานโครงข่ายประสาทเทียมมีตั้งแต่การใช้เพื่อตัดสินใจง่ายไปจนถึงงาน ที่มีความยุ่งยากซับซ้อน ตัวอย่างการประยุกต์ใช้งานบางส่วน ได้แก่ งานด้านการควบคุม งานด้านการบิน ด้านยานยนต์ ด้านการบริหารจัดการ ด้านการธนาคาร ด้านการทหาร ด้านการบันเทิง และ อื่น ๆ อีกมากมาย โครงข่ายประสาทเทียมมีประวัติความเป็นมาย้อนหลังไปประมาณ 60 กว่าปีก่อน ในปี ค.ศ. 1943 Mc Culloch และ Pitts แห่งมหาวิทยาลัยชิคาโก ประเทศสหรัฐอเมริกา ได้นำเสนอ บทความวิชาการ “Boolean brain” ซึ่งได้กลายเป็นจุดกำเนิดของการจัดรูปแบบคณิตศาสตร์ของ ประสาทเทียม ต่อมาได้มีนักวิจัยได้คิดค้นรูปแบบโครงข่ายประสาทเทียมแบบต่าง ๆ มากมาย และ ทุกรูปแบบวิธีจะประกอบกับวิธีการสอนโครงข่ายด้วย ซึ่งวิธีการต่าง ๆ จะมีความซับซ้อนแตกต่างกันไป โครงข่ายประสาทเทียม (artificial neural network: ANN) เป็นการจำลองการทำงานของโครงข่ายประสาทของมนุษย์ (Biological Neurons) ซึ่งประกอบด้วยส่วนของการประมวลผลที่ เรียกว่านิวรอน (Neuron) ทุก ๆ นิวรอนสามารถมีอินพุตได้หลายอินพุตแต่มีเอาต์พุตเพียงเอาต์พุต เดียว และทุก ๆ เอาต์พุตจะ

แยกไปยังอินพุตของนิวรอนอื่นๆ ภายในโครงข่าย การติดต่อกันภายในระหว่างนิวรอนไม่ใช่ลักษณะการต่อแบบธรรมดาทุก ๆ อินพุตจะมีน้ำหนักเป็นตัวกำหนดกำลังของการติดต่อภายในและช่วยในการตัดสินใจ การทำงานของนิวรอนในบางโครงข่ายจะถูกกำหนดไว้ตายตัว แต่บางโครงข่ายสามารถที่จะปรับแต่งได้ซึ่งอาจจะเป็นการปรับแต่งจากภายนอก โครงข่ายหรือนิวรอนสามารถปรับได้ด้วยตัวเอง ในจุดนี้แสดงถึงความสามารถในการเรียนรู้และ จดจำของโครงข่ายประสาทเทียม สมองประกอบด้วยประสาทจำนวนมหาศาล (ประมาณ 1011) และมีจุดต่อจำนวนโครงข่ายประสาทประกอบขึ้นด้วยส่วนสำคัญ 3 ส่วน คือ โยประสาท (nerve fiber หรือ dendrites) ตัวเซลล์ (cell body หรือ soma) และแกนประสาทนำออก (axon) ในแต่ละโครงข่ายประสาทจะเชื่อมต่อกันโดยจุดประสานประสาท (synapse) ซึ่งสามารถเปลี่ยนค่าความต้านทานได้ตามสัญญาณที่ส่งระหว่างกันของเซลล์ประสาท การส่งสัญญาณระหว่างเซลล์ประสาททำได้โดยการถ่ายเทสารประกอบไอออนและโพแทสเซียม

โครงข่ายร่างแหระบบประสาทเทียม (neural network) หรือที่มักจะเรียกสั้น ๆ ว่า ข่ายงานประสาท (neural network หรือ neural net) คือโมเดลทางคณิตศาสตร์ สำหรับประมวลผลสารสนเทศด้วยการคำนวณแบบคอนเนกชันนิสต์ (connectionist) เพื่อจำลองการทำงานของเครือข่ายประสาทในสมองมนุษย์ด้วยวัตถุประสงค์ที่จะวางเครื่องมือซึ่งมีความสามารถในการเรียนรู้การจดจำแบบรูป (Pattern Recognition) และการอุปมาความรู้เช่นเดียวกับความสามารถที่มีในสมองมนุษย์ แนวคิดเริ่มต้นของเทคนิคนี้ได้มาจากการศึกษาข่ายงานไฟฟ้าชีวภาพ (bioelectric network) ในสมอง ซึ่งประกอบด้วย เซลล์ประสาท หรือ “นิวรอน” (neurons) และ จุดประสานประสาท (synapses) แต่ละเซลล์ประสาทประกอบด้วยปลายในการรับกระแสประสาท เรียกว่า “เดนไดรต์” (Dendrite) ซึ่งเป็น input และปลายในการส่งกระแสประสาทเรียกว่า “แอกซอน” (Axon) ซึ่งเป็นเหมือน output ของเซลล์ เซลล์เหล่านี้ทำงานด้วยปฏิกิริยาไฟฟ้าเคมี เมื่อมีการกระตุ้นด้วยสิ่งเร้าภายนอกหรือกระตุ้นด้วยเซลล์ด้วยกัน กระแสประสาทจะวิ่งผ่านเดนไดรต์เข้าสู่นิวเคลียสซึ่งจะเป็นตัวตัดสินใจว่าต้องกระตุ้นเซลล์อื่นๆ ต่อหรือไม่ ถ้ากระแสประสาทแรงพอ นิวเคลียสก็จะกระตุ้นเซลล์อื่นๆ ต่อไปผ่านทางแอกซอนของมัน

แบบจำลองโครงข่ายร่างแหระบบประสาทเทียม ดังระบบสมการต่อไปนี้

$$\dot{x}(t) = -Ax(t) + BS(x(t-h)) + f$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbf{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่ายทอด

B เป็นเมทริกซ์ถ่วงน้ำหนัก

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

และ

$$x'(t) = -Ax(t) + BS(x(t-h)) + u(t) + f$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbf{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่ายทอด

B เป็นเมทริกซ์ถ่วงน้ำหนัก

$u(t) \in \mathbf{R}^n$ เป็นตัวควบคุม

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

วัตถุประสงค์ของการวิจัย

ในงานวิจัยนี้ เราได้ศึกษาเงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและการหาค่าเหมาะที่สุดของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม แบบจำลองโครงข่ายร่างแหระบบประสาทเทียม ดังระบบสมการต่อไปนี้

$$x'(t) = -Ax(t) + BS(x(t-h)) + f$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbf{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่ายทอด

B เป็นเมทริกซ์ถ่วงน้ำหนัก

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

ประโยชน์ที่คาดว่าจะได้รับ

ได้เงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม และมีโปรแกรมคำนวณทางคณิตศาสตร์(MATLAB) ของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม ซึ่งสามารถนำไปใช้ประโยชน์ในทางการแพทย์และชีววิทยา เพื่อสร้างแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม และนำไปประยุกต์ใช้ในการสร้างระบบ

ประสาทเทียมในคนพิการและอัมพาตได้ในอนาคตต่อไป และเผยแพร่ผลงานวิจัยการประชุมวิชาการระดับนานาชาติ และตีพิมพ์ผลงานวิจัยในวารสารระดับนานาชาติ

การตรวจเอกสาร

1. ค้นคว้างานวิจัยที่เกี่ยวข้อง จัดเตรียมหาข้อมูลที่เกี่ยวข้องของการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท
2. หาเงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท
3. นำเงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาทนำมาเปรียบเทียบกับตัวอย่างเชิงตัวเลข โดยเขียนโปรแกรมคำนวณทางคณิตศาสตร์ (MATLAB)
4. เรียบเรียงงานวิจัยและส่งตีพิมพ์เผยแพร่ในระดับชาติและนานาชาติ

พิจารณาสมการ

$$\dot{x} = f(t, x) \quad (*)$$

โดยที่ $x = (x_1, x_2, \dots, x_N)^T \in R^n, x_i = x_i(t), f = (f_1, f_2, \dots, f_N)^T \in R^n, f_i = f_i(t, x_1, x_2, \dots, x_N)$ สำหรับ $i = 1, 2, \dots, N$

บทนิยาม 1 จุดสมดุลของสมการ (*) คือ $x = a$ ที่ทำให้ $f(t, a) = 0$

บทนิยาม 2 กำหนดให้ $\bar{0}$ เป็นจุดสมดุลของสมการ (*) แล้วจะกล่าวว่า จุดสมดุล

- 1) เสถียร (stable) สำหรับทุก $\varepsilon > 0$ มี $\delta > 0$ ซึ่ง $\|x(t_0)\| < \delta$ แล้ว $\|x(t)\| < \varepsilon$ สำหรับทุก $t \geq t_0$
- 2) เสถียรเชิงเส้นกำกับ (asymptotically stable) ถ้า $\bar{0}$ เป็นจุดสมดุลที่เสถียร และ $\|x(t)\| \rightarrow 0$ เมื่อ $t \rightarrow +\infty$
- 3) ไม่เสถียร (unstable) ถ้าไม่เป็นไปตาม 1) นั่นคือ มี $\varepsilon > 0$ ทุก $\delta > 0$ ซึ่ง $\|x(t_0)\| < \delta$ และ $\|x(t_0)\| \geq \varepsilon$ สำหรับบาง $t \geq t_0$

บทนิยาม 3 กำหนดให้ P เป็นเมทริกซ์ค่าจริง ถ้า $P = P^T$ แล้วจะกล่าวว่า P เป็นเมทริกซ์สมมาตร (Symmetric matrix)

บทนิยาม 4 กำหนดให้ P เป็นเมทริกซ์สมมาตร จะกล่าวว่า

- 1) P เป็น เมทริกซ์บวกแน่นอน (Positive definite) ก็ต่อเมื่อ $x^T P x > 0$ สำหรับ
ทุกๆ $x \in R^n$ และ $x \neq \bar{0}$
- 2) P เป็น เมทริกซ์กึ่งบวกแน่นอน (Positive semi definite) ก็ต่อเมื่อ $x^T P x \geq 0$ สำหรับ
ทุกๆ $x \in R^n$ และ $x^T P x = 0$ เมื่อ $x = \bar{0}$
- 3) P เป็น เมทริกซ์ลบแน่นอน (Negative definite) ก็ต่อเมื่อ $x^T P x < 0$ สำหรับ
ทุกๆ $x \in R^n$ และ $x \neq \bar{0}$
- 4) P เป็น เมทริกซ์กึ่งลบแน่นอน (Negative semi definite) ก็ต่อเมื่อ $x^T P x \leq 0$ สำหรับ
ทุกๆ $x \in R^n$ และ $x^T P x = 0$ เมื่อ $x = \bar{0}$

บทนิยาม 5 ฟังก์ชันไลปูนอฟ (Lyapunov function)

กำหนดให้ $V: R^n \rightarrow R$ จะกล่าวว่า $V(x(t))$ เป็นฟังก์ชันไลปูนอฟของสมการ (*) ถ้า $V(x(t))$ สอดคล้องกับเงื่อนไขต่อไปนี้

- 1) $V(x(t))$ เป็นฟังก์ชันต่อเนื่อง บน R^n
- 2) $V(x(t))$ เป็นฟังก์ชันบวกแน่นอน นั่นคือ $V(x(t)) > 0$ สำหรับ $x(t) \neq \bar{0}$ และ $V(\bar{0}) = 0$
- 3) อนุพันธ์ย่อยของ $V(x(t))$ เกี่ยวกับ t

$$\dot{V}(x(t)) = \frac{\partial V}{\partial x_1} \dot{x}_1 + \frac{\partial V}{\partial x_2} \dot{x}_2 + \dots + \frac{\partial V}{\partial x_N} \dot{x}_N = \frac{\partial V}{\partial x_1} f_1 + \frac{\partial V}{\partial x_2} f_2 + \dots + \frac{\partial V}{\partial x_N} f_N$$

เป็นฟังก์ชันกึ่งลบแน่นอน นั่นคือ $\dot{V}(x(t)) \leq 0$ สำหรับ $x(t) \neq \bar{0}$ และ $\dot{V}(\bar{0}) = 0$

ทฤษฎีบท 6 กำหนดให้ $x = \bar{0}$ เป็นจุดสมดุลของสมการ (*) จะได้ว่า

- 1) จุดสมดุลของสมการ (*) จะเสถียรถ้าฟังก์ชันไลปูนอฟ $V(x(t))$ ที่สอดคล้องกับบท
นิยาม 5
- 2) จุดสมดุลของสมการ (*) จะเสถียรเชิงเส้นกำกับ ถ้าฟังก์ชันไลปูนอฟ $V(x(t))$ ที่
สอดคล้องกับบทนิยามที่ 5 และอนุพันธ์ย่อยของ $V(x(t))$ เป็นลบแน่นอน นั่นคือ
 $\dot{V}(x(t)) < 0$ สำหรับ $x(t) \neq \bar{0}$ และ $\dot{V}(\bar{0}) = 0$

ทฤษฎีบท 7 ให้ $P \in R^{n \times n}$ เป็นเมทริกซ์สมมาตร กำหนดให้ $\lambda_{\min}(P)$ เป็น ค่าลักษณะเฉพาะที่มีค่าน้อยที่สุดของเมทริกซ์ P และ $\lambda_{\max}(P)$ เป็นค่า ลักษณะเฉพาะที่มีค่ามากที่สุดของเมทริกซ์ P แล้ว
 $\lambda_{\min}(P)x^T x \leq x^T P x \leq \lambda_{\max}(P)x^T x$ สำหรับ $x \in R^{n \times n}$

ทฤษฎีบท 8 กำหนดให้ P เป็นเมทริกซ์สมมาตรจะกล่าวว่า

- 1) P เป็นเมทริกซ์ลบแน่นอน (Negative definite) ก็ต่อเมื่อ ทุกค่าลักษณะเฉพาะ (eigenvalue) เป็นลบแท้จริง (strictly negative)
- 2) P เป็นเมทริกซ์บวกแน่นอน (Positive definite) ก็ต่อเมื่อ ทุกค่าลักษณะเฉพาะ (eigenvalue) เป็นบวกแท้จริง (strictly positive)

กำหนดสัญลักษณ์

R^+ - เซตของจำนวนจริงบวก

R^n - ปริภูมิ n มิติของเวกเตอร์ค่าจริง

$R^{n \times r}$ - เมทริกซ์ค่าจริงขนาด $n \times r$ มิติ

$\lambda(A)$ - เซตของค่าลักษณะเฉพาะของเมทริกซ์ A

$\lambda_{\max}(A) = \max\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\}$

$\lambda_{\min}(A) = \min\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\}$

$\Lambda \geq 0$ - A เป็นเมทริกซ์กึ่งบวกแน่นอน

บทนิยาม 9 ให้ $\beta > 0$ ระบบสมการจะเสถียรแบบเลขชี้กำลังที่มีอัตราการรู้เข้า β ถ้ามีฟังก์ชันการสลับ $\sigma(\cdot)$ และมีจำนวนบวก γ ที่ซึ่งทุกๆ ผลเฉลย $x(t, \phi)$ ของระบบสมการสอดคล้องกับ $\|x(t, \phi)\| \leq \gamma e^{-\beta t} \|\phi\| \quad \forall t \in R^+$

บทนิยาม 10 จะกล่าวว่าระบบของเมทริกซ์ $\{L_i\}, i \in \{1, 2, \dots, N\}$ strictly complete

ถ้าทุกๆ $x \in R^n \setminus \{0\}$ มี $i \in \{1, 2, \dots, N\}$ ที่ซึ่ง $x^T L_i x < 0$ กำหนดให้ $\Omega_i = \{x \in R^n : x^T L_i x < 0\}$
 $i = 1, 2, \dots, N$

ทฤษฎีบท 11 ระบบของเมทริกซ์ $\{L_i\}, i \in \{1, 2, \dots, N\}$ strictly complete ก็ต่อเมื่อ

$$\bigcup_{i=1}^N \Omega_i = R^n \setminus \{0\}$$

ทฤษฎีบท 12 เงื่อนไขเพียงพอสำหรับระบบเมทริกซ์ $\{L_i\}, i \in \{1, 2, \dots, N\}$ strictly complete

ถ้ามี $\xi_i \geq 0, i \in \{1, 2, \dots, N\}$ และ $\sum_{i=1}^N \xi_i > 0$ ที่ซึ่ง $\sum_{i=1}^N \xi_i L_i < 0$

ถ้า $N = 2$ เงื่อนไขข้างต้นจะเป็นเงื่อนไขจำเป็นสำหรับ strictly complete ด้วย

บทตั้ง 13 สำหรับทุกๆ $x, y \in \mathbb{R}^n$ และเมทริกซ์ Y, E, F, H โดยที่ $Y > 0, F^T F \leq I$ และสเกลาร์ $\varepsilon > 0$ จะได้สอสมการต่อไปนี้เป็นจริง

$$1) EFH + H^T F^T E^T \leq \varepsilon^{-1} EE^T + \varepsilon H^T H$$

$$2) 2x^T y \leq x^T Y^{-1} x + y^T Y y$$

อุปกรณ์และวิธีการวิจัย

1. รวบรวมเอกสารงานวิจัยทั้งหมด หนังสือและบทความทั้งหมดในวารสารต่างๆที่เกี่ยวข้องกับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท
2. ศึกษาวิธีการต่างๆสำหรับการออกแบบฟังก์ชันไลปูนอฟ สำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท
3. การใช้ความรู้พื้นฐานและวิธีการต่างๆข้อ 1 และ 2 เพื่อการศึกษาและสร้างวิธีการใหม่สำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท
4. พิสูจน์ทฤษฎีของการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท
5. เขียนงานวิจัยและส่งไปตีพิมพ์ในวารสารระดับนานาชาติ

ผลการวิจัย

พิจารณาระบบควบคุมเชิงผลต่างของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม ดังระบบสมการต่อไปนี้

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k) \quad (1)$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbb{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่ายทอด

B เป็นเมทริกซ์ถ่วงน้ำหนัก

$u(t) \in \mathbb{R}^n$ เป็นตัวควบคุม

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

สำหรับการออกแบบตัวควบคุม ตอนนี้เรามีความสนใจการออกแบบตัวควบคุมสำหรับระบบ (1) ดังนี้

$$u(k) = Kx(k)$$

รูปแบบใหม่ของ (1) ดังต่อไปนี้

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + DKx(k) \quad (2)$$

ทฤษฎีบท 1 จุดสมดุลที่ $x = 0$ ของ (2) เป็นเสถียรภาพเชิงเส้นกำกับ ถ้ามี P, G, W, R เป็นเมตริกซ์บวกแน่นอน และสอดคล้องสมการเมตริกซ์

$$\psi = \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} < 0 \quad (3)$$

ซึ่งมี

$$(1,1) = CPC + CPDK + K^T D^T PC + K^T D^T PDK - P + CPAL + L^T A^T PC \\ + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W$$

$$(1,2) = CPBL + K^T D^T PBL + L^T A^T PBL$$

$$(2,1) = L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL$$

$$(2,2) = L^T B^T PBL - G - \hat{h}R$$

$$\hat{h} = h_2 - h_1 + 1$$

พิสูจน์ เราจะใช้ฟังก์ชันไลยาปูนอฟดังนี้

$$V_1(x(k)) = x^T(k)Px(k)$$

$$V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i)$$

$$V_3(x(k)) = \sum_{j=k-h_2+1}^{k-h_1} \sum_{i=j}^{k-1} x^T(i)Wx(i)$$

$$V_4(x(k)) = \sum_{i=k-h(k)}^{k-1} (h(k)-k+i)x^T(i)Rx(i)$$

หาอนุพันธ์ของ ตามเงื่อนไขของ (2) จะได้

$$\begin{aligned}
\Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k)) \\
&= [Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k)]^T \\
&\quad \times P[Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k)] \\
&\quad - x^T(k)Px(k) \\
&= x^T(k)[CPC + CPDK + K^T D^T PC + K^T D^T PDK - P]x(k) \\
&\quad + x^T(k)CPAS(x(k)) + S^T(x(k))A^T PCx(k) \\
&\quad + x^T(k)CPBS(x(k-h(k))) + S^T(x(k-h(k)))B^T PCx(k)
\end{aligned}$$

$$\begin{aligned}
\Delta V &\leq x^T(k)[CPC + CPDK + K^T D^T PC + K^T D^T PDK - P \\
&\quad + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W]x(k) \\
&\quad + x^T(k)[CPBL + K^T D^T PBL + L^T A^T PBL]x(k-h(k)) \\
&\quad + x^T(k-h(k))[L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL]x(k) \\
&\quad + x^T(k-h(k))[L^T B^T PBL - G - \hat{h}R]x(k-h(k)), \\
&= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}^T \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix} \\
&= y^T(k) \psi y(k)
\end{aligned}$$

ซึ่งมี

$$\begin{aligned}
(1,1) &= CPC + CPDK + K^T D^T PC + K^T D^T PDK - P \\
&\quad + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W \\
(1,2) &= CPBL + K^T D^T PBL + L^T A^T PBL \\
(2,1) &= L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL \\
(2,2) &= L^T B^T PBL - G - \hat{h}R \\
y(k) &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}
\end{aligned}$$

โดยนิยามที่ 5 ระบบสลับที่มีตัวควบคุม (2) จะมีเสถียรเชิงเส้นกำกับ

พิจารณาระบบเชิงผลต่างของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียมที่มีตัวหน่วงหลายตัว
 ดังระบบสมการต่อไปนี้

$$u(k+1) = -Au(k) + \sum_{i=1}^m B_i S(u(k-h_i)) \quad (4)$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbf{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่วงทอด

B_i , $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่วงน้ำหนัก

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

ทฤษฎีบท 2 จุดสมดุลที่ $x = 0$ ของ (4) เป็นเสถียรภาพเชิงเส้นกำกับ ถ้ามี P, G_i, W_i ,

$i = 1, 2, \dots, m$, และ $L = \text{diag}[l_1, \dots, l_n] > 0$ เป็นเมทริกซ์บวกแน่นอน และสอดคล้องสมการเมตริกซ์

$$\psi = \begin{bmatrix} (0,0) & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & (1,1) & (1,2) & \dots & (1,m) & 0 & 0 & 0 & \dots & 0 \\ 0 & (2,1) & (2,2) & \dots & (2,m) & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & (m,1) & (m,2) & \dots & (m,m) & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & (m+1,m+1) & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (m+2,m+2) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & (2m,2m) \end{bmatrix} < 0 \quad (5)$$

ซึ่งมี

$$(0,0) = A^T P A - P + \sum_{i=1}^m (h_i G_i + W_i) + \varepsilon \sum_{i=1}^m \sum_{j=1}^m A^T P B_i B_j^T P A,$$

$$(1,1) = \varepsilon^{-1} L L + L B_1^T P B_1 L - W_1,$$

$$(1,2) = \varepsilon^{-1} L L + L B_1^T P B_2 L,$$

$$(1,m) = \varepsilon^{-1} L L + L B_1^T P B_m L,$$

$$(2,1) = \varepsilon^{-1} L L + L B_2^T P B_1 L,$$

$$(2,2) = \varepsilon^{-1} L L + L B_2^T P B_2 L - W_2,$$

$$(2,m) = \varepsilon^{-1} L L + L B_2^T P B_m L,$$

$$(m,1) = \varepsilon^{-1} L L + L B_m^T P B_1 L,$$

$$(m,2) = \varepsilon^{-1} L L + L B_m^T P B_2 L,$$

$$(m,m) = \varepsilon^{-1} L L + L B_m^T P B_m L - W_m,$$

$$(m+1,m+1) = -h_1 G_1,$$

$$(m+2,m+2) = -h_2 G_2,$$

$$(2m, 2m) = -h_m G_m.$$

พิสูจน์ เราจะใช้ฟังก์ชันไลยาปูนอฟดังนี้

$$V_1(y(k)) = x^T(k)Px(k),$$

$$V_2(y(k)) = \sum_{i=1}^m \sum_{j=k-h_i+1}^k (h-k+i)x^T(j)G_i x(j),$$

$$V_3(y(k)) = \sum_{i=1}^m \sum_{j=k-h_i+1}^k x^T(j)W_i x(j),$$

หาอนุพันธ์ของ ตามเงื่อนไขของ (4) จะได้

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)),$$

$$\Delta V_1(y(k)) = V_1(x(k+1)) - V_1(x(k))$$

$$= [-Ax(k) + \sum_{i=1}^m B_i S(x(k-h_i))]^T P [-Ax(k) + \sum_{i=1}^m B_i S(x(k-h_i))] - x^T(k)Px(k)$$

$$= x^T(k)[A^T PA - P]x(k)$$

$$- \sum_{i=1}^m x^T(k)A^T PB_i S(x(k-h_i))$$

$$- \sum_{i=1}^m S^T(x(k-h_i))B_i^T PAx(k)$$

$$+ \sum_{i=1}^m \sum_{j=1}^m S^T(x(k-h_i))B_i^T PB_j S(x(k-h_j)),$$

$$\Delta V_2(y(k)) = \Delta \left(\sum_{i=1}^m \sum_{j=k-h_i+1}^k (h_i - k + j)x^T(j)G_i x(j) \right)$$

$$= \sum_{i=1}^m h_i x^T(k)G_i x(k) - \sum_{i=1}^m \sum_{j=k-h_i+1}^k x^T(j)G_i x(j),$$

$$\begin{aligned}\Delta V_3(y(k)) &= \Delta \left(\sum_{i=1}^m \sum_{j=k-h_i+1}^k x^T(j) W_i x(j) \right) \\ &= \sum_{i=1}^m x^T(k) W_i x(k) - \sum_{i=1}^m x^T(k-h_i) W_i x(k-h_i)\end{aligned}$$

โดยนิยามที่ 5 ระบบสลับที่มีตัวควบคุม (4) จะมีเสถียรเชิงเส้นกำกับ

พิจารณาระบบเชิงผลต่างของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม ดังระบบสมการต่อไปนี้

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) \quad (6)$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbf{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่ายทอด

B เป็นเมทริกซ์ถ่วงน้ำหนัก

$u(t) \in \mathbf{R}^n$ เป็นตัวควบคุม

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

ทฤษฎีบท 3 จุดสมดุลที่ $x = 0$ ของ (6) เป็นเสถียรภาพเชิงเส้นกำกับ ถ้ามี P, G, W, R เป็นเมตริกซ์บวกแน่นอน และสอดคล้องอสมการเมตริกซ์

$$\psi = \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} < 0 \quad (7)$$

ซึ่งมี

$$(1,1) = CPC - P + CPAL + L^T A^T PC + L^T A^T PAL + \hat{h}W$$

$$(1,2) = CPBL + L^T A^T PBL$$

$$(2,1) = L^T B^T PC + L^T B^T PAL$$

$$(2,2) = L^T B^T PBL - G - \hat{h}R$$

$$\hat{h} = h_2 - h_1 + 1$$

พิสูจน์ เราจะใช้ฟังก์ชันไลยาปูนอฟดังนี้

$$V_1(x(k)) = x^T(k)Px(k)$$

$$V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i)$$

$$V_3(x(k)) = \sum_{j=k-h_2+1}^{k-h_1} \sum_{i=j}^{k-1} x^T(i)Wx(i)$$

$$V_4(x(k)) = \sum_{i=k-h(k)}^{k-1} (h(k)-k+i)x^T(i)Rx(i)$$

หาอนุพันธ์ของ ตามเงื่อนไขของ (6) จะได้

$$\begin{aligned} \Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [Cx(k) + AS(x(k)) + BS(x(k-h(k)))]^T \\ &\quad \times P[Cx(k) + AS(x(k)) + BS(x(k-h(k)))] \\ &\quad - x^T(k)Px(k) \\ &= x^T(k)[CPC - P]x(k) \\ &\quad + x^T(k)CPAS(x(k)) + S^T(x(k))A^T PCx(k) \\ &\quad + x^T(k)CPBS(x(k-h(k))) + S^T(x(k-h(k)))B^T PCx(k) \end{aligned}$$

$$\begin{aligned} \Delta V &\leq x^T(k)[CPC - P \\ &\quad + CPAL + L^T A^T PC + L^T A^T PAL + \hat{h}W]x(k) \\ &\quad + x^T(k)[CPBL + L^T A^T PBL]x(k-h(k)) \\ &\quad + x^T(k-h(k))[L^T B^T PC + L^T B^T PAL]x(k) \\ &\quad + x^T(k-h(k))[L^T B^T PBL - G - \hat{h}R]x(k-h(k)), \\ &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}^T \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix} \\ &= y^T(k)\psi y(k) \end{aligned}$$

ซึ่งมี

$$(1,1) = CPC - P + CPAL + L^T A^T PC + L^T A^T PAL + \hat{h}W$$

$$(1,2) = CPBL + L^T A^T PBL$$

$$(2,1) = L^T B^T PC + L^T B^T PAL$$

$$(2,2) = L^T B^T PBL - G - \hat{h}R$$

$$y(k) = \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}$$

โดยนิยามที่ 5 ระบบสลับที่มีตัวควบคุม (6) จะมีเสถียรเชิงเส้นกำกับ

พิจารณาระบบเชิงผลต่างของแบบจำลองโครงข่ายร่างแหระบบประสาทเทียม ดังระบบสมการต่อไปนี้

$$x(k+1) = -Ax(k) + BS(x(k-h)) + Cu(k) \quad (8)$$

ซึ่งมี

$x(t) \in \Omega \subseteq \mathbf{R}^n$ เป็นสถานะของระบบประสาท

$A = \text{diag}\{a_1, \dots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \dots, n$ เป็นเมทริกซ์ถ่ายทอด

B, C, K เป็นเมทริกซ์ถ่วงน้ำหนัก

$u(t) \in \mathbf{R}^n$ เป็นตัวควบคุม

$S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ ซึ่งมี s_i เป็น ตัวกระตุ้นระบบประสาท

สำหรับการออกแบบตัวควบคุม ตอนนี้เราให้ความสนใจการออกแบบตัวควบคุมสำหรับระบบ (8) ดังนี้

$$u(k) = Kx(k)$$

รูปแบบใหม่ของ (8) ดังต่อไปนี้

$$x(k+1) = -Ax(k) + BS(x(k-h)) + CKx(k) \quad (9)$$

ทฤษฎีบท 4 จุดสมดุลที่ $x = 0$ ของ (9) เป็นเสถียรภาพเชิงเส้นกำกับ ถ้ามี P, G, W และ $L = \text{diag}[l_1, \dots, l_n] > 0$ เป็นเมทริกซ์บวกแน่นอน และสอดคล้องสมการเมทริกซ์

$$\psi = \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} < 0 \quad (10)$$

ซึ่งมี

$$\begin{aligned} (1,1) &= APA - APCK - K^T C^T PA - C^T K^T PC - P + hG + W \\ &\quad + \varepsilon APBB^T PA + \varepsilon_1 K^T C^T PBB^T PCK, \\ (2,2) &= \varepsilon^{-1} LL + \varepsilon_1^{-1} LL + LB^T PBL - W, \\ (3,3) &= -hG. \end{aligned}$$

พิสูจน์ เราจะใช้ฟังก์ชันไลยาปูนอฟดังนี้

$$V_1(y(k)) = x^T(k)Px(k),$$

$$V_2(y(k)) = \sum_{i=k-h+1}^k (h-k+i)x^T(i)Gx(i),$$

$$V_3(y(k)) = \sum_{i=k-h+1}^k x^T(i)Wx(i),$$

หาอนุพันธ์ของ ตามเงื่อนไขของ (9) จะได้

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)),$$

$$\begin{aligned} \Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [-Ax(k) + BS(x(k-h)) + CKx(k)]^T P [-Ax(k) + BS(x(k-h)) + CKx(k)] \\ &\quad - x^T(k)Px(k) \end{aligned}$$

$$\begin{aligned} &= x^T(k)[APA - APCK - K^T C^T PA - C^T K^T PC - P]x(k) \\ &\quad - x^T(k)APBS(x(k-h)) - S^T(x(k-h))B^T P Ax(k) \\ &\quad + x^T(k)K^T C^T PBS(x(k-h)) + S^T(x(k-h))B^T PCKx(k) \\ &\quad + S^T(x(k-h))B^T PBS(x(k-h)), \end{aligned}$$

$$\Delta V_2(y(k)) = \Delta \left(\sum_{i=k-h+1}^k (h-k+i)x^T(i)Gx(i) \right) = hx^T(k)Gx(k) - \sum_{i=k-h+1}^k x^T(i)Gx(i),$$

$$\Delta V_3(y(k)) = \Delta \left(\sum_{i=k-h+1}^k x^T(i)Wx(i) \right) = x^T(k)Wx(k) - x^T(k-h)Wx(k-h),$$

$$-x^T(k)APBS(x(k-h)) - S^T(x(k-h))B^T P Ax(k) \leq \varepsilon x^T(k)APBB^T P Ax(k)$$

$$+ \varepsilon^{-1} S^T(x(k-h))S(x(k-h)),$$

$$\begin{aligned} x^T(k)K^T C^T PBS(x(k-h)) + S^T(x(k-h))B^T PCKx(k) &\leq \varepsilon_1 x^T(k)K^T C^T PBB^T PCKx(k) \\ &\quad + \varepsilon_1^{-1} S^T(x(k-h))S(x(k-h)), \end{aligned}$$

$$S^T(x(k-h))B^T PBS(x(k-h)) \leq x^T(k-h)LB^T PBLx(k-h),$$

$$\varepsilon^{-1} S^T(x(k-h))S(x(k-h)) \leq \varepsilon^{-1} x^T(k-h)LLx(k-h),$$

$$\varepsilon_1^{-1} S^T(x(k-h))S(x(k-h)) \leq \varepsilon_1^{-1} x^T(k-h)LLx(k-h),$$

$$\Delta V_1(y(k)) \leq x^T(k)[A^T PA - P]x(k) + \varepsilon x^T(k)A^T PBB^T PAx(k) \\ + \varepsilon^{-1}x^T(k-h)LLx(k-h) + x^T(k-h)LB^T PBLx(k-h).$$

$$\Delta V \leq x^T(k)[APA - APCK - K^T C^T PA - C^T K^T PC - P + hG + W \\ + \varepsilon APBB^T PA + \varepsilon_1 K^T C^T PBB^T PCK]x(k) \\ + x^T(k-h)[\varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^T PBL - W]x(k-h) \\ - \sum_{i=k-h}^{k-1} x^T(i)Gx(i).$$

$$\sum_{i=k-h+1}^k x^T(i)Gx(i) \geq \left(\frac{1}{h} \sum_{i=k-h+1}^k x(i) \right)^T (hG) \left(\frac{1}{h} \sum_{i=k-h+1}^k x(i) \right).$$

$$\Delta V \leq x^T(k)[APA - APCK - K^T C^T PA - C^T K^T PC - P + hG + W \\ + \varepsilon APBB^T PA + \varepsilon_1 K^T C^T PBB^T PCK]x(k) \\ + x^T(k-h)[\varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^T PBL - W]x(k-h) \\ - \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right)^T (hG) \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right) \\ = \begin{pmatrix} x^T(k), x^T(k-h), \left(\frac{1}{h} \sum_{i=k-h+1}^k x(i) \right)^T \end{pmatrix} \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h) \\ \left(\frac{1}{h} \sum_{i=k-h+1}^k x(i) \right) \end{pmatrix} \\ = y^T(k) \Psi y(k),$$

$$(1,1) = APA - APCK - K^T C^T PA - C^T K^T PC - P + hG + W \\ + \varepsilon APBB^T PA + \varepsilon_1 K^T C^T PBB^T PCK,$$

$$(2,2) = \varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^T PBL - W,$$

$$(3,3) = -hG,$$

$$y(k) = \begin{pmatrix} x(k) \\ x(k-h) \\ \left(\frac{1}{h} \sum_{i=k-h+1}^k x(i) \right) \end{pmatrix}.$$

โดยนิยามที่ 5 ระบบสลับที่มีตัวควบคุม (6) จะมีเสถียรเชิงเส้นกำกับ

วิจารณ์ผลการวิจัย

งานวิจัยนี้ได้นำเสนอการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท โดยการประยุกต์ใช้ฟังก์ชันไลปูนอฟในการแก้ปัญหามีเสถียรภาพและค่าเหมาะที่สุดและพัฒนาเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์ จากกรณีนี้เป็นเงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาทที่จะนำเสนอการปรับปรุงพัฒนาผลลัพธ์ของการมีเสถียรภาพ และค่าเหมาะที่สุดของแบบจำลองระบบประสาทจะนำเสนอในตัวอย่าง เนื่องจากเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์ แล้วพบว่าเงื่อนไขที่ได้นั้นยังมีความซับซ้อนมาก ดังนั้นจึงควรศึกษาตัวแปรอื่นที่มีความสัมพันธ์เกี่ยวข้องเพิ่มเติมในการศึกษาวิจัยครั้งต่อไป ควรมีการศึกษาปัจจัยด้านอื่นๆที่เกี่ยวข้องอันจะเป็นประโยชน์ในการวิเคราะห์ และนำมาใช้ในสร้างเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์

สรุปผลการวิจัย

งานวิจัยนี้ได้นำเสนอการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาท โดยการประยุกต์ใช้ฟังก์ชันไลปูนอฟในการแก้ปัญหามีเสถียรภาพและค่าเหมาะที่สุดและพัฒนาเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์ จากกรณีนี้เป็นเงื่อนไขที่เพียงพอสำหรับการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาทที่จะนำเสนอการปรับปรุงพัฒนาผลลัพธ์ของการมีเสถียรภาพและค่าเหมาะที่สุดของแบบจำลองระบบประสาทจะนำเสนอในตัวอย่าง เนื่องจากเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์ แล้วพบว่าเงื่อนไขที่ได้นั้นยังมีความซับซ้อนมาก ดังนั้นจึงควรศึกษาตัวแปรอื่นที่มีความสัมพันธ์เกี่ยวข้องเพิ่มเติม ในการศึกษาวิจัยครั้งต่อไป ควรมีการศึกษาปัจจัยด้านอื่น ๆ ที่เกี่ยวข้องอันจะเป็นประโยชน์ในการวิเคราะห์ และนำมาใช้ในสร้างเงื่อนไขความมีเสถียรภาพที่ขึ้นอยู่กับตัวหน่วงเวลาที่ได้มาในรูปของอสมการเมทริกซ์

การเผยแพร่ผลงานวิจัยการประชุมวิชาการระดับนานาชาติดังนี้

1. Delay-dependent Asymptotical Stabilization Criterion of Recurrent Neural Networks, Grienggrai Rajchakit, the 2013 4th International Conference on Mechanical, Industrial, and Manufacturing Technologies (MIMT 2013), Bali Island, Indonesia, March 16-17, 2013.

2. Guaranteed cost control for Hopfield neural networks with interval non differentiable time-varying delay, Grienggrai Rajchakit, International Conference on Applied Analysis and Mathematical Modelling (ICAAMM 2013) , Istanbul-Turkey, 2-5 June 2013.

การเสนองานวิจัยแบบปากเปล่าในการประชุมวิชาการระดับนานาชาติดังนี้

1. Delay-dependent Asymptotical Stabilization Criterion of Recurrent Neural Networks, Grienggrai Rajchakit, the 2013 4th International Conference on Mechanical, Industrial, and Manufacturing Technologies (MIMT 2013), Bali Island, Indonesia, March 16-17, 2013.

(ISI, SCOPUS)

2. Guaranteed cost control for Hopfield neural networks with interval non differentiable time-varying delay, Grienggrai Rajchakit, International Conference on Applied Analysis and Mathematical Modelling (ICAAMM 2013) , Istanbul-Turkey.

3. New design switching rule for the robust mean square stability of uncertain stochastic discrete-time hybrid systems, Grienggrai Rajchakit, 2013 Third World Congress on Information and Communication Technologies (WICT), Hanoi, Vietnam 15-18 December, 2013.

(IEEE, SCOPUS)

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การตีพิมพ์ในระดับนานาชาติดังนี้

1. Grienggrai Rajchakit, DELAY-DEPENDENT ASYMPTOTICAL STABILIZATION CRITERION OF RECURRENT NEURAL NETWORKS, Applied Mechanics and Materials, Vol. 330 (2013) pp 1045-1048 Online available since 2013/Jun/27 at www.scientific.net © (2013) Trans Tech Publications, Switzerland doi:10.4028/www.scientific.net/AMM.330.1045 (ISI)

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Delay-dependent asymptotical stabilization criterion of recurrent neural networks

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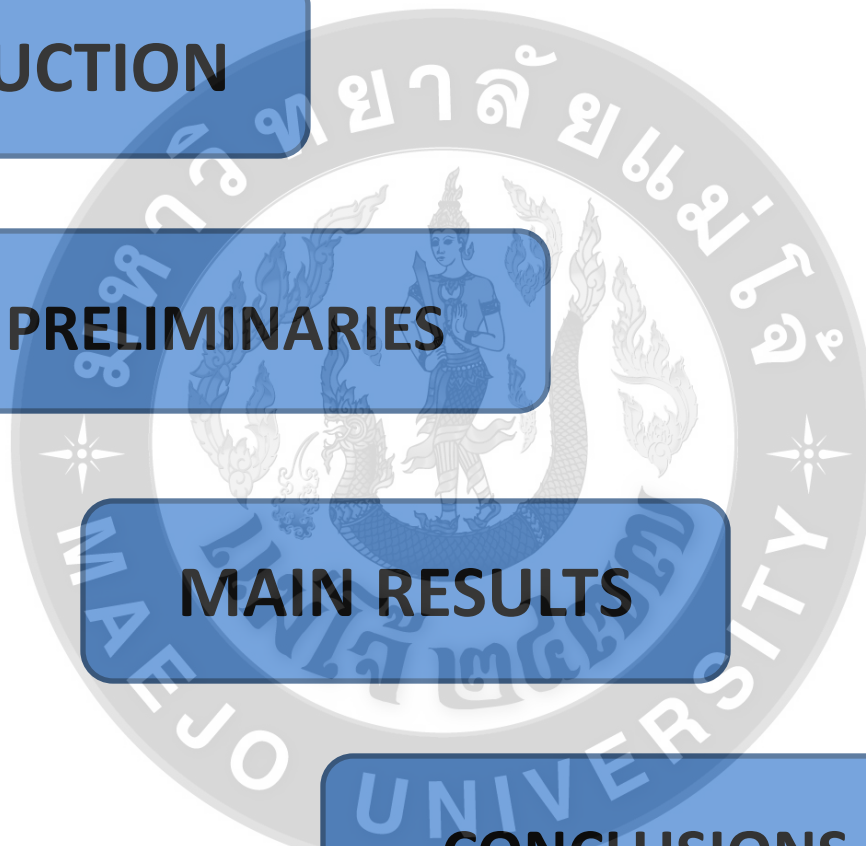


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INTRODUCTION

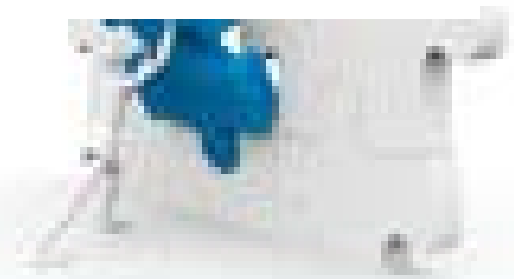
A recurrent neural network (RNNs) is a very important tool for many application areas such as associative memory, pattern recognition, signal processing, model identification and combinatorial optimization. With the development of research on RNNs in theory and application, the model is more and more complex. Parameter uncertainties and nonautonomous phenomena often exist in real systems due to modeling inaccuracies [1, 2]. Particularly when we consider a longterm dynamical behavior of the system and consider seasonality of the changing environment, the parameters of the system usually will change with time [3, 4]. Simultaneously, in implementations of artificial neural networks, time delay may occur due to finite switching speeds of the amplifiers and communication time [5, 6]. In order to model those systems with neural networks, the neural networks with time-varying delay appear in many papers [7, 8]. So in this paper we consider the stability of the following discrete-time recurrent neural networks:

In this paper, we consider control discrete-time system of neural networks of the form

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$$v(k+1) = Cv(k) + AS(v(k)) + BS(v(k-h(k))) + Du(k) + f, \quad (1)$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $0 < h_1 \leq h(k) \leq h_2, \forall k = 0, 1, 2, \dots$, $C = \text{diag}\{c_1, \dots, c_n\}$, $c_i \geq 0$, $i = 1, 2, \dots, n$ is the $n \times n$ constant relaxation matrix, A, B are the $n \times n$ constant weight matrix, D is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector, $f = (f_1, \dots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1, 1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, \dots, n$.

The asymptotic stability of the zero solution of the delay-differential system of Hopfield neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the control discrete-time system of neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.



PRELIMINARIES

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all non-negative real numbers; $\mathbb{Z}^+$ denotes the set of all non-negative integers; $\mathbb{R}^n$ denotes the n -finite-dimensional Euclidean space with the Euclidean norm $\|\cdot\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbb{R}^{n \times m}$ denotes the set of all $(n \times m)$ -matrices; and A^T denotes the transpose of the matrix A ; Matrix $Q \in \mathbb{R}^{n \times n}$ is positive semidefinite ($Q \geq 0$) if $x^T Q x \geq 0$, for all $x \in \mathbb{R}^n$.

If $x^T Q x > 0$ ($x^T Q x < 0$, resp.) for any $x \neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0$, ($Q < 0$, resp.). It is easy to verify that $Q > 0$, ($Q < 0$, resp.)

Iff $\exists \beta > 0: x^T Q x \geq \beta \|x\|^2, \forall x \in \mathbb{R}^n$,

Iff $\exists \beta > 0: x^T Q x \geq \beta \|x\|^2, \forall x \in \mathbb{R}^n$,

($\exists \beta > 0: x^T Q x \leq -\beta \|x\|^2, \forall x \in \mathbb{R}^n$, resp.).



Lemma 1. [1] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}^+$ such that

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\beta \|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is asymptotically stable.

MAIN RESULTS

In this section, we consider the sufficient condition for asymptotic stability of the zero solution v^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0$, $S(0) = 0$ and $f = 0$ (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*)$).

The new form of (1) is now given by

$$\begin{aligned} x(k+1) = & Cx(k) + AS(x(k)) + BS(x(k-h(k))) \\ & + Du(k). \end{aligned} \quad (2)$$

This is a basic requirement for controller design. Now, we are interested designing a feedback controller for the system (2) as

$$u(k) = Kx(k).$$

where K is $n \times m$ constant control gain matrix.



The new form of (2) is now given by

$$\begin{aligned} x(k+1) = & Cx(k) + AS(x(k)) + BS(x(k-h(k))) \\ & + DKx(k). \end{aligned} \quad (3)$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i = 1, 2, \dots, n$ is bounded and monotonically nondecreasing on $\mathbb{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i > 0, i = 1, 2, \dots, n$ such that

$$|s_i(r_1) - s_i(r_2)| \leq l_i |r_1 - r_2|, \quad \forall r_1, r_2 \in \mathbb{R}. \quad (4)$$

By condition (4), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \leq l_i |x_i|, \quad i = 1, 2, \dots, n. \quad (5)$$

Theorem 1. The zero solution of the control discrete-time system of neural networks (3) is asymptotically stable if there exist symmetric positive definite matrices P, G, W, R satisfying the following matrix inequalities of the form

$$\psi = \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} < 0, \quad (6)$$

where

$$(1,1) = CPC + CPDK + K^T D^T PC + K^T D^T PDK - P + CPAL \\ + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W,$$

$$(1,2) = CPBL + K^T D^T PBL + L^T A^T PBL,$$

$$(2,1) = L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL,$$

$$(2,2) = L^T B^T PBL - G - \hat{h}R, \text{ and } \hat{h} = h_2 - h_1 + 1.$$

Proof. Consider the Lyapunov function candidate,

where

$$V_1(x(k)) = x^T(k)Px(k),$$

$$V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i),$$

$$V_3(x(k)) = \sum_{j=k-h_2+1}^{k-h_1} \sum_{i=j}^{k-1} x^T(i)Wx(i),$$

$$V_4(x(k)) = \sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Rx(i).$$



The Lyapunov difference of the system along trajectory of solution of (3) is given by

$$\begin{aligned}\Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [Cx(k) + AS(x(k)) \\ &\quad + BS(x(k-h(k))) + Du(k)]^T \\ &\quad \times P[Cx(k) + AS(x(k)) \\ &\quad + BS(x(k-h(k))) + Du(k)] \\ &\quad - x^T(k)Px(k)\end{aligned}$$



$$\begin{aligned}
&= x^T(k)[CPC + CPDK + K^T D^T PC \\
&\quad + K^T D^T PDK - P]x(k) \\
&\quad + x^T(k)CPAS(x(k)) \\
&\quad + S^T(x(k))A^T PCx(k) \\
&\quad + x^T(k)CPBS(x(k-h(k))) \\
&\quad + S^T(x(k-h(k)))B^T PCx(k) \\
&\quad + x^T(k)K^T D^T PAS(x(k)) \\
&\quad + S^T(x(k))A^T PDKx(k) \\
&\quad + x^T(k)K^T D^T PBS(x(k-h(k))) \\
&\quad + S^T(x(k-h(k)))B^T PDKx(k) \\
&\quad + S^T(x(k))A^T PBS(x(k-h(k))) \\
&\quad + S^T(x(k-h(k)))B^T PAS(x(k)) \\
&\quad + S^T(x(k))A^T PAS(x(k)) \\
&\quad + S^T(x(k-h(k)))B^T PBS(x(k-h(k))).
\end{aligned}$$



Based on (5), we obtain

$$\begin{aligned}
 \Delta V_1(x(k)) \leq & x^T(k) [CPC + CPDK \\
 & + K^T D^T PC + K^T D^T PDK \\
 & - P] x(k) + x^T(k) CPALx(k) \\
 & + x^T(k) L^T A^T PCx(k) \\
 & + x^T(k) CPBLx(k-h(k)) \\
 & + x^T(k-h(k)) L^T B^T PCx(k) \\
 & + x^T(k) K^T D^T PALx(k) \\
 & + x^T(k) L^T A^T PDKx(k) \\
 & + x^T(k) K^T D^T PBLx(k-h(k)) \\
 & + x^T(k-h(k)) L^T B^T PDKx(k) \\
 & + x^T(k) L^T A^T PBLx(k-h(k)) \\
 & + x^T(k-h(k)) L^T B^T PALx(k) \\
 & + x^T(k) L^T A^T PALx(k) \\
 & + x^T(k-h(k)) L^T B^T PBLx(k-h(k)).
 \end{aligned}$$



Therefore,

$$\begin{aligned} \Delta V_1 \leq & x^T(k) [CPC + CPDK \\ & + K^T D^T PC + K^T D^T PDK - P \\ & + CPAL + L^T A^T PC + K^T D^T PAL \\ & + L^T A^T PDK + L^T A^T PAL] x(k) \\ & + x^T(k) [CPBL + K^T D^T PBL \\ & + L^T A^T PBL] x(k - h(k)) \\ & + x^T(k - h(k)) [L^T B^T PC \\ & + L^T B^T PDK x(k) + L^T B^T PAL] x(k) \\ & x^T(k - h(k)) [L^T B^T PBL] x(k - h(k)), \end{aligned}$$

and, we get

$$\begin{aligned}
 \Delta V_2(x(k)) &= \sum_{i=k+1-h(k+1)}^k x^T(i)Gx(i) - \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i) \\
 &= \sum_{i=k-h(k+1)}^{k-h_1} x^T(i)Gx(i) + x^T(k)Gx(k) \\
 &\quad - x^T(k-h(k))Gx(k-h(k)) \\
 &\quad + \sum_{k+1-h_1}^{k-1} x^T(k)Gx(k) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Gx(i).
 \end{aligned}$$

Since $h(k) \geq h_1$, we get

$$\sum_{k+1-h_1}^{k-1} x^T(k)Gx(k) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Gx(i) \leq 0.$$

Therefore,

$$\Delta V_2(x(k)) \leq \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Gx(i) + x^T(k)Gx(k) - x^T(k-h(k))Gx(k-h(k)).$$

Similarly, we obtain

$$\begin{aligned} \Delta V_3(x(k)) &= \sum_{j=k-h_2+1}^{k-h_1} \left[x^T(k)Wx(k) + \sum_{i=k+j}^{k-1} x^T(i)Wx(i) \right] \\ &\quad - \sum_{j=k-h_2+1}^{k-h_1} \left[\sum_{i=k+j-1}^{k-1} x^T(i)Wx(i) \right] \\ &= (h_2 - h_1)x^T(k)Wx(k) - \sum_{i=k+1-h_2}^{k-h_1} x^T(i)Wx(i). \end{aligned}$$

$$\begin{aligned} \Delta V_4(k, y(k)) &= \sum_{i=k+1-h(k+1)}^{k+1-1} (h(k+1) - k + 1 + i)x^T(i)Rx(i) \\ &\quad - \sum_{i=k+1-h(k+1)}^{k+1-1} (h(k) - k + i)x^T(i)Rx(i) \end{aligned}$$



$$\begin{aligned}
 &= (h_2 - h_1) \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i) R x(i) \\
 &\quad + (h_2 - h_1) x^T(k) R x(k) \\
 &\quad - (h_2 - h_1) x^T(k - h(k)) R x(k - h(k)) \\
 &\quad + (h_2 - h_1) \sum_{k+1-h_1}^{k-1} x^T(k) R x(k) \\
 &\quad - (h_2 - h_1) \sum_{i=k+1-h(k)}^{k-1} x^T(i) R x(i).
 \end{aligned}$$

Since $h(k) \geq h_1$, we get

$$(h_2 - h_1) \sum_{k+1-h_1}^{k-1} x^T(k) R x(k) - (h_2 - h_1) \sum_{i=k+1-h(k)}^{k-1} x^T(i) R x(i) \leq 0.$$



Therefore,

$$\begin{aligned}\Delta V_4(k, y(k)) &= (h_2 - h_1) \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i) R x(i) \\ &\quad + (h_2 - h_1) x^T(k) R x(k) \\ &\quad - (h_2 - h_1) x^T(k - h(k)) R x(k - h(k)).\end{aligned}$$

From $\Delta V_2(x(k))$, $\Delta V_3(x(k))$, and $\Delta V_4(x(k))$, we get

$$\begin{aligned}\Delta V_2(x(k)) + \Delta V_3(x(k)) + \Delta V_4(x(k)) &\leq \\ &\sum_{i=k-h(k+1)}^{k-h_1} x^T(i) G x(i) + x^T(k) G x(k) \\ &\quad - x^T(k - h(k)) G x(k - h(k)) \\ &\quad + (h_2 - h_1) \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i) R x(i) \\ &\quad + (h_2 - h_1) x^T(k) R x(k) \\ &\quad - (h_2 - h_1) x^T(k - h(k)) R x(k - h(k)) \\ &\quad + \hat{h} x^T(k) W x(k) - \sum_{i=k+1-h_2}^{k-h_1} x^T(i) W x(i).\end{aligned}$$



Where $\hat{h} = h_2 - h_1 + 1$. Since $h(k) \leq h_2$, we have

$$\begin{aligned} & \sum_{i=k-h(k+1)}^{k-h_1} x^T(i)Gx(i) + x^T(k)Gx(k) \\ & + (h_2 - h_1) \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Rx(i) \\ & + (h_2 - h_1)x^T(k)Rx(k) \\ & - \sum_{i=k+1-h_2}^{k-h_1} x^T(i)Wx(i) \leq 0. \end{aligned}$$

Thus,

$$\begin{aligned} & \Delta V_2(x(k)) + \Delta V_3(x(k)) + \Delta V_4(x(k)) \leq \\ & \hat{h} x^T(k)Wx(k) - x^T(k-h(k))Gx(k-h(k)) \\ & - \hat{h} x^T(k-h(k))Rx(k-h(k)). \end{aligned}$$



As a result, we obtain

$$\begin{aligned}
 \Delta V &\leq x^T(k)[CPC + CPDK + K^T D^T PC \\
 &\quad + K^T D^T PDK - P + CPAL + L^T A^T PC \\
 &\quad + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL \\
 &\quad + \hat{h}W]x(k) + x^T(k)[CPBL + K^T D^T PBL \\
 &\quad + L^T A^T PBL]x(k-h(k)) \\
 &\quad + x^T(k-h(k))[L^T B^T PC + L^T B^T PDKx(k) \\
 &\quad + L^T B^T PAL]x(k) \\
 &\quad + x^T(k-h(k))[L^T B^T PBL - G - \hat{h}R]x(k-h(k)) \\
 &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}^T \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix} \\
 &= y^T(k)\psi y(k),
 \end{aligned}$$



Where,

$$(1,1) = CPC + CPDK + K^T D^T PC + K^T D^T PDK \\ - P + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK \\ + L^T A^T PAL + \hat{h}W,$$

$$(1,2) = CPBL + K^T D^T PBL + L^T A^T PBL,$$

$$(2,1) = L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL,$$

$$(2,2) = L^T B^T PBL - G - \hat{h}R,$$

$$y(k) = \begin{pmatrix} x(k) \\ x(k - h(k)) \end{pmatrix}.$$

By the condition (6), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 1**. This completes the proof. $\square$

Example 1. Let us consider a control discrete-time system of neural networks (3), given by the system

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + DKx(k),$$

where the matrices are

$$C = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.8 \end{pmatrix}, A = \begin{pmatrix} 0.5 & -0.2 \\ 0 & -0.8 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.4 & -0.1 \\ 0 & -0.9 \end{pmatrix}, L = \begin{pmatrix} 0.3 & 0.1 \\ 0 & 0.2 \end{pmatrix},$$

$$D = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \varepsilon = 0.3, \varepsilon_1 = 0.3.$$



Using the LMI Toolbox in MATLAB, we found that the LMIs in **Theorem 1** are feasible and

$$P = \begin{pmatrix} 24.3226 & -2.0016 \\ -2.0016 & 15.2293 \end{pmatrix}, G = \begin{pmatrix} 6.3397 & -1.2698 \\ -1.2698 & 3.2248 \end{pmatrix}$$
$$W = \begin{pmatrix} 7.3396 & -2.3369 \\ -2.3369 & 4.2697 \end{pmatrix}, R = \begin{pmatrix} 2.6914 & -0.9672 \\ -0.9672 & 3.2737 \end{pmatrix},$$

$0.2379 \leq h(k) \leq 14.6692$ are set of solutions to the LMIs (6).

By a straightforward, we have

$$\psi = \begin{pmatrix} -2.3579 & 0 \\ 0 & -1.6942 \end{pmatrix}.$$

The eigenvalues are -2.3579 and -1.6942, respectively. This implies the matrix $\psi < 0$. It follows from **Lemma 1** that the zero solution of control discrete-time system of neural networks is asymptotically stable.

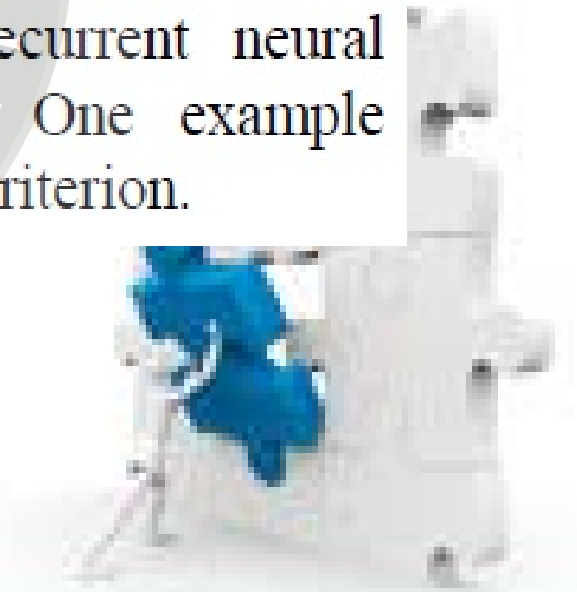


Remark 1. **Theorem 1** gives a sufficient condition for the asymptotic stability of control discrete-time system of neural networks (3) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed. But [9, 10] these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed.



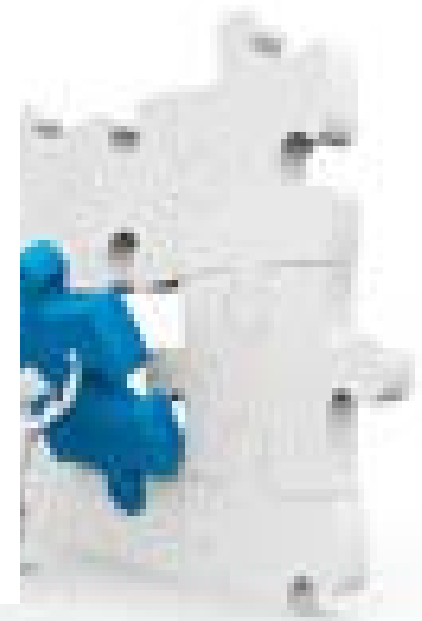
CONCLUSIONS

This paper was dedicated to the delay-dependent stability of discrete-time recurrent neural networks with time-varying delay. A less conservative LMI-based globally stability criterion is obtained with quadratic Lyapunov functional approach and free-weighting matrix approach for periodic discrete-time recurrent neural networks with a time-varying delay. One example illustrates the exactness of the proposed criterion.



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Dear Dr. Ratchagit,

It is my pleasure to invite you to visit our Institute from June 19-26, 2013. During the visit you are invited to collaborate with Prof. Vu Ngoc Phat's research group at the Department of Control and Optimization.

Your accommodation and living expenses shall be financed by the Department of Control and Optimization. We hope very much that you will be able to accept our invitation.

I am looking forward to seeing you in Hanoi.

Sincerely yours

Vu N. Phat

RESEARCH

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Guaranteed cost control for switched recurrent neural networks with interval time-varying delay

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Abstract

This paper studies the problem of guaranteed cost control for a class of switched recurrent neural networks with interval time-varying delay. The time delay is a continuous function belonging to a given interval, but not necessary differentiable. A cost function is considered as a nonlinear performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of augmented Lyapunov-Krasovskii functionals, a guaranteed cost controller is designed via memoryless state feedback control, a switching rule for the exponential stabilization for the system is designed via linear matrix inequalities and new sufficient conditions for the existence of the guaranteed cost state-feedback for the system are given in terms of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the obtained result.

Keywords: neural networks; guaranteed cost control; switching design; stabilization; interval time-varying delays; Lyapunov function; linear matrix inequalities

1 Introduction

Stability and control of recurrent neural networks with time delay have attracted considerable attention in recent years [1–8]. In many practical systems, it is desirable to design neural networks which are not only asymptotically or exponentially stable but can also guarantee an adequate level of system performance. In the area of control, signal processing, pattern recognition and image processing, delayed neural networks have many useful applications. Some of these applications require that the equilibrium points of the designed network be stable. In both biological and artificial neural systems, time delays due to integration and communication are ubiquitous and often become a source of instability. The time delays in electronic neural networks are usually time-varying, and sometimes vary violently with respect to time due to the finite switching speed of amplifiers and faults in the electrical circuitry. Guaranteed cost control problem [9–12] has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound. The Lyapunov-Krasovskii functional technique has been among the popular and effective tools in the design of guaranteed cost controls for neural networks with time delay. Nevertheless, despite such a diversity of results available, the most existing works either assumed that the time delays are constant or differentiable [13–16].

Although, in some cases, delay-dependent guaranteed cost control for systems with time-varying delays were considered in [12, 13, 15], the approach used there cannot be applied to systems with interval, non-differentiable time-varying delays. To the best of our knowledge, the guaranteed cost control and state feedback stabilization for switched recurrent neural networks with interval time-varying delay, non-differentiable time-varying delays have not been fully studied yet (see, e.g., [9–12, 15–25] and the references therein). Which are important in both theories and applications. This motivates our research.

In this paper, we investigate the guaranteed cost control for switched recurrent neural networks problem. The novel features here are that the delayed neural network under consideration is with various globally Lipschitz continuous activation functions, and the time-varying delay function is interval, non-differentiable. Specifically, our goal is to develop a constructive way to design a switching rule to exponentially stabilize the system. A nonlinear cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. Based on constructing a set of augmented Lyapunov-Krasovskii functionals combined with the Newton-Leibniz formula, new delay-dependent criteria for guaranteed cost control via memoryless feedback control are established in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing MATLABs LMI control toolbox.

The outline of the paper is as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main result. LMI delay-dependent criteria for guaranteed cost control and a numerical example showing the effectiveness of the result are presented in Section 3. The paper ends with conclusions and cited references.

2 Preliminaries

The following notation will be used in this paper. $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the n -dimensional space with the scalar product $\langle x, y \rangle$ or $x^T y$ of two vectors x, y , and the vector norm $\| \cdot \|$; $M^{n \times r}$ denotes the space of all matrices of $(n \times r)$ dimensions. A^T denotes the transpose of matrix A ; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A ; $\lambda_{\max}(A) = \max\{\operatorname{Re} \lambda; \lambda \in \lambda(A)\}$. $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\|x_t\| = \sup_{s \in [-h, 0]} \|x(t+s)\|$; $C^1([0, t], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$ -valued continuously differentiable functions on $[0, t]$; $L_2([0, t], \mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$ -valued square integrable functions on $[0, t]$.

Matrix A is called semi-positive definite ($A \geq 0$) if $\langle Ax, x \rangle \geq 0$ for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $\langle Ax, x \rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. The notation $\operatorname{diag}\{\dots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following switched recurrent neural networks with interval time-varying delay:

$$\begin{aligned} \dot{x}(t) &= -A_{\gamma(x(t))}x(t) + W_{0\gamma(x(t))}f(x(t)) \\ &\quad + W_{1\gamma(x(t))}g(x(t-h(t))) + B_{\gamma(x(t))}u(t), \quad t \geq 0, \\ x(t) &= \phi(t), \quad t \in [-h_1, 0], \end{aligned} \quad (2.1)$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state of the neural, $u(\cdot) \in L_2([0, t], \mathbb{R}^m)$ is the control; n is the number of neurons, and

$$\begin{aligned} f(x(t)) &= [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T, \\ g(x(t)) &= [g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t))]^T, \end{aligned}$$

are the activation functions; $\gamma(\cdot) : \mathbb{R}^n \rightarrow \mathcal{N} := \{1, 2, \dots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = j$ implies that the system realization is chosen as the j th system, $j = 1, 2, \dots, N$. It is seen that system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries.

$A_j = \text{diag}(\bar{a}_{1j}, \bar{a}_{2j}, \dots, \bar{a}_{nj})$, $\bar{a}_{ij} > 0$, represents the self-feedback term; $B_j \in \mathbb{R}^{n \times m}$ are control input matrices; W_{0j} , W_{1j} denote the connection weights and the delayed connection weights, respectively. The time-varying delay function $h(t)$ satisfies the condition

$$0 \leq h_0 \leq h(t) \leq h_1.$$

The initial functions $\phi(t) \in C^1([-h_1, 0], \mathbb{R}^n)$, with the norm

$$\|\phi\| = \sup_{t \in [-h_1, 0]} \sqrt{\|\phi(t)\|^2 + \|\dot{\phi}(t)\|^2}.$$

In this paper we consider various activation functions and assume that the activation functions $f(\cdot)$, $g(\cdot)$ are Lipschitzian with the Lipschitz constants $f_i, e_i > 0$:

$$\begin{aligned} |f_i(\xi_1) - f_i(\xi_2)| &\leq f_i |\xi_1 - \xi_2|, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in \mathbb{R}, \\ |g_i(\xi_1) - g_i(\xi_2)| &\leq e_i |\xi_1 - \xi_2|, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in \mathbb{R}. \end{aligned} \quad (2.2)$$

The performance index associated with system (2.1) is the following function:

$$J = \int_0^\infty f^0(t, x(t), x(t-h(t)), u(t)) dt, \quad (2.3)$$

where $f^0(t, x(t), x(t-h(t)), u(t)) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^+$, is a nonlinear cost function satisfying

$$\exists Q_1, Q_2, \quad R : f^0(t, x, y, u) \leq \langle Q_1 x, x \rangle + \langle Q_2 y, y \rangle + \langle Ru, u \rangle \quad (2.4)$$

for all $(t, x, y, u) \in \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$ and $Q_1, Q_2 \in \mathbb{R}^{n \times n}$, $R \in \mathbb{R}^{m \times m}$, are given symmetric positive definite matrices. The objective of this paper is to design a memoryless state feedback controller $u(t) = Kx(t)$ for system (2.1) and the cost function (2.3) such that the resulting closed-loop system

$$\dot{x}(t) = -(A_j - B_j K)x(t) + W_{0j}f(x(t)) + W_{1j}g(x(t-h(t))) \quad (2.5)$$

is exponentially stable and the closed-loop value of the cost function (2.3) is minimized.

Remark 2.1 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero; therefore, the stability criteria proposed in [4–7, 9–13, 15–18, 21–24] are not applicable to this system.

Remark 2.2 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the delay function $h(t)$ is non-differentiable; therefore, the stability criteria proposed in [5, 6, 8, 10–12, 14–19, 22–25] are not applicable to this system.

Definition 2.1 Given $\alpha > 0$. The zero solution of closed-loop system (2.5) is α -exponentially stabilizable if there exists a positive number $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\|x(t, \phi)\| \leq Ne^{-\alpha t} \|\phi\|, \quad \forall t \geq 0.$$

Definition 2.2 Consider control system (2.1). If there exist a memoryless state feedback control law $u(t) = Kx(t)$ and a positive number J^* such that the zero solution of closed-loop system (2.5) is exponentially stable and the cost function (2.3) satisfies $J \leq J^*$, then the value J^* is a guaranteed constant and $u(t)$ is a guaranteed cost control law of the system and its corresponding cost function.

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.1 (Schur complement lemma [26]) *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

Proposition 2.2 (Integral matrix inequality [27]) *For any symmetric positive definite matrix $M > 0$, scalar $\sigma > 0$ and vector function $\omega : [0, \sigma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^\sigma \omega(s) ds \right)^T M \left(\int_0^\sigma \omega(s) ds \right) \leq \sigma \left(\int_0^\sigma \omega^T(s) M \omega(s) ds \right).$$

3 Design of guaranteed cost controller

In this section, we give a design of memoryless guaranteed feedback cost control for neural networks (2.1). Let us set

$$w_{11} = -[P + \alpha I]A_j - A_j^T[P + \alpha I] - 2B_j B_j^T + 0.25B_j R B_j^T + \sum_{i=0}^1 G_i,$$

$$w_{12} = P + A_j P + 0.5B_j B_j^T,$$

$$w_{13} = e^{-2\alpha h_0} H_0 + 0.5B_j B_j^T + A_j P,$$

$$w_{14} = 2e^{-2\alpha h_1} H_1 + 0.5B_j B_j^T + A_j P,$$

$$\begin{aligned}
 w_{15} &= P0.5B_jB_j^T + A_jP, \\
 w_{22} &= \sum_{i=0}^1 W_{ij}D_iW_{ij}^T + \sum_{i=0}^1 h_i^2H_i + (h_1 - h_0)U - 2P - B_jB_j^T, \\
 w_{23} &= P, \quad w_{24} = P, \quad w_{25} = P, \\
 w_{33} &= -e^{-2\alpha h_0}G_0 - e^{-2\alpha h_0}H_0 - e^{-2\alpha h_1}U + \sum_{i=0}^1 W_{ij}D_iW_{ij}^T, \\
 w_{34} &= 0, \quad w_{35} = -2\alpha h_1U, \\
 w_{44} &= \sum_{i=0}^1 W_{ij}D_iW_{ij}^T - e^{-2\alpha h_1}U - e^{-2\alpha h_1}G_1 - e^{-2\alpha h_1}H_1, \quad w_{45} = e^{-2\alpha h_1}U, \\
 w_{55} &= -e^{-2\alpha h_1}U + W_{0j}D_0W_{0j}^T, \\
 E &= \text{diag}\{e_i, i = 1, \dots, n\}, \quad F = \text{diag}\{f_i, i = 1, \dots, n\}, \\
 \lambda_1 &= \lambda_{\min}(P^{-1}), \\
 \lambda_2 &= \lambda_{\max}(P^{-1}) + h_0\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^1 G_i\right)P^{-1}\right] \\
 &\quad + h_1^2\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^1 H_i\right)P^{-1}\right] + (h_1 - h_0)\lambda_{\max}(P^{-1}UP^{-1}).
 \end{aligned}$$

Theorem 3.1 Consider control system (2.1) and the cost function (2.3). If there exist symmetric positive definite matrices P, U, G_0, G_1, H_0, H_1 and diagonal positive definite matrices $D_i, i = 0, 1$, satisfying the following LMIs:

$$\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix} < 0, \quad j = 1, 2, \dots, N, \quad (3.1)$$

$$\mathcal{S}_{1j} = \begin{bmatrix} -PA_j - A_j^T P - \sum_{i=0}^1 e^{-2\alpha h_i} H_i & 2PF & PQ_1 \\ * & -D_0 & 0 \\ * & * & -Q_1^{-1} \end{bmatrix} < 0, \quad j = 1, 2, \dots, N, \quad (3.2)$$

$$\mathcal{S}_{2j} = \begin{bmatrix} W_{1j}D_1W_{1j}^T - e^{-2\alpha h_1}U & 2PE & PQ_2 \\ * & -D_1 & 0 \\ * & * & -Q_2^{-1} \end{bmatrix} < 0, \quad j = 1, 2, \dots, N, \quad (3.3)$$

then

$$u_j(t) = -\frac{1}{2}B_j^T P^{-1}x(t), \quad t \geq 0, j = 1, 2, \dots, N, \quad (3.4)$$

is a guaranteed cost control and the guaranteed cost value is given by

$$J^* = \lambda_2 \|\phi\|^2.$$

The switching rule is chosen as $\gamma(x(t)) = j$. Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad \forall t \geq 0.$$

Proof Let $Y = P^{-1}$, $y(t) = Yx(t)$. Using the feedback control (2.5), we consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t, x_t) &= \sum_{i=1}^6 V_i(t, x_t), \\ V_1 &= x^T(t) Y x(t), \\ V_2 &= \int_{t-h_0}^t e^{2\alpha(s-t)} x^T(s) Y G_0 Y x(s) ds, \\ V_3 &= \int_{t-h_1}^t e^{2\alpha(s-t)} x^T(s) Y G_1 Y x(s) ds, \\ V_4 &= h_0 \int_{-h_0}^0 \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y H_0 Y \dot{x}(\tau) d\tau ds, \\ V_5 &= h_1 \int_{-h_1}^0 \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y H_1 Y \dot{x}(\tau) d\tau ds, \\ V_6 &= (h_1 - h_0) \int_{t-h_1}^{t-h_0} \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y U Y \dot{x}(\tau) d\tau ds. \end{aligned}$$

It is easy to check that

$$\lambda_1 \|x(t)\|^2 \leq V(t, x_t) \leq \lambda_2 \|x_t\|^2, \quad \forall t \geq 0. \quad (3.5)$$

Taking the derivative of V_1 , we have

$$\begin{aligned} \dot{V}_1 &= 2x^T(t) Y \dot{x}(t) \\ &= y^T(t) [-PA_j^T - A_j P] y(t) - y^T(t) B_j B_j^T y(t) \\ &\quad + 2y^T(t) W_{0j} f(\cdot) y(t) + 2y^T(t) W_{1j} g(\cdot) y(t) \\ \dot{V}_2 &= y^T(t) G_0 y(t) - e^{-2\alpha h_0} y^T(t-h_0) G_0 y(t-h_0) - 2\alpha V_2; \\ \dot{V}_3 &= y^T(t) G_1 y(t) - e^{-2\alpha h_1} y^T(t-h_1) G_1 y(t-h_1) - 2\alpha V_3; \\ \dot{V}_4 &= h_0^2 \dot{y}^T(t) H_0 \dot{y}(t) - h_1 e^{-2\alpha h_0} \int_{t-h_0}^t \dot{x}^T(s) H_0 \dot{x}(s) ds - 2\alpha V_4; \\ \dot{V}_5 &= h_1^2 \dot{y}^T(t) H_1 \dot{y}(t) - h_1 e^{-2\alpha h_1} \int_{t-h_1}^t \dot{y}^T(s) H_1 \dot{y}(s) ds - 2\alpha V_4; \\ \dot{V}_6 &= (h_1 - h_0)^2 \dot{y}^T(t) U \dot{y}(t) - (h_1 - h_0) e^{-2\alpha h_1} \int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds - 2\alpha V_6. \end{aligned}$$

Applying Proposition 2.2 and the Leibniz-Newton formula

$$\int_s^t \dot{y}(\tau) d\tau = y(t) - y(s),$$

we have, for $j = 1, 2$, $i = 0, 1$,

$$\begin{aligned} -h_i \int_{t-h_i}^t \dot{y}^T(s) H_j \dot{y}(s) ds &\leq - \left[\int_{t-h_i}^t \dot{y}(s) ds \right]^T H_j \left[\int_{t-h_i}^t \dot{y}(s) ds \right] \\ &\leq -[y(t) - y(t-h(t))]^T H_j [y(t) - y(t-h(t))] \\ &= -y^T(t) H_i y(t) + 2x^T(t) H_j y(t-h(t)) \\ &\quad - y^T(t-h_i) H_j y(t-h_i). \end{aligned} \quad (3.6)$$

Note that

$$\int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds = \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds + \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds.$$

Applying Proposition 2.2 gives

$$\begin{aligned} [h_1 - h(t)] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds &\geq \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s) ds \right]^T U \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s) ds \right] \\ &\geq [y(t-h(t)) - y(t-h_1)]^T U [y(t-h(t)) - y(t-h_1)]. \end{aligned}$$

Since $h_1 - h(t) \leq h_1 - h_0$, we have

$$[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds \geq [y(t-h(t)) - y(t-h_1)]^T U [y(t-h(t)) - y(t-h_1)],$$

then

$$-[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds \leq -[y(t-h(t)) - y(t-h_1)]^T U [y(t-h(t)) - y(t-h_1)].$$

Similarly, we have

$$-(h_1 - h_0) \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds \leq -[y(t-h_0) - y(t-h(t))]^T U [y(t-h_0) - y(t-h(t))].$$

Then we have

$$\begin{aligned} \dot{V}(\cdot) + 2\alpha V(\cdot) &\leq y^T(t) [-PA_j^T - A_j P] y(t) - y^T(t) B_j B_j^T y(t) + 2y^T(t) W_0 f(\cdot) \\ &\quad + 2y^T(t) W_1 g(\cdot) + y^T(t) \left(\sum_{i=0}^1 G_i \right) y(t) + 2\alpha \langle Py(t), y(t) \rangle \\ &\quad + \dot{y}^T(t) \left(\sum_{i=0}^1 h_i^2 H_i \right) \dot{y}(t) + (h_1 - h_0) \dot{y}^T(t) U \dot{y}(t) \end{aligned}$$

$$\begin{aligned}
& - \sum_{i=0}^1 e^{-2\alpha h_i} y^T(t-h_i) G_i y(t-h_i) \\
& - e^{-2\alpha h_0} [y(t) - y(t-h_0)]^T H_0 [y(t) - y(t-h_0)] \\
& - e^{-2\alpha h_1} [y(t) - y(t-h_1)]^T H_1 [y(t) - y(t-h_1)] \\
& - e^{-2\alpha h_1} [y(t-h(t)) - y(t-h_1)]^T U [y(t-h(t)) - y(t-h_1)] \\
& - e^{-2\alpha h_1} [y(t-h_0) - y(t-h(t))]^T U [y(t-h_0) - y(t-h(t))]. \quad (3.7)
\end{aligned}$$

Using equation (2.5)

$$P\dot{y}(t) + A_j P y(t) - W_{0j} f(\cdot) - W_{1j} g(\cdot) + 0.5 B_j B_j^T y(t) = 0,$$

and multiplying both sides by $[2y^T(t), -2\dot{y}^T(t), 2y^T(t-h_0), 2y^T(t-h_1), 2y^T(t-h(t))]$, we have

$$\begin{aligned}
& 2y^T(t) P \dot{y}(t) + 2y^T(t) A_j P y(t) - 2y^T(t) W_{0j} f(\cdot) - 2y^T(t) W_{1j} g(\cdot) \\
& + y^T(t) B_j B_j^T y(t) = 0, \\
& -2\dot{y}^T(t) P \dot{y}(t) - 2\dot{y}^T(t) A_j P y(t) + 2\dot{y}^T(t) W_{0j} f(\cdot) \\
& + 2\dot{y}^T(t) W_{1j} g(\cdot) - \dot{y}^T(t) B_j B_j^T y(t) = 0, \\
& 2y^T(t-h_0) P \dot{y}(t) + 2y^T(t-h_0) A_j P y(t) - 2y^T(t-h_0) W_{0j} f(\cdot) \\
& - 2y^T(t-h_0) W_{1j} g(\cdot) + y^T(t-h_0) B_j B_j^T y(t) = 0, \\
& 2y^T(t-h_1) P \dot{y}(t) + 2y^T(t-h_1) A_j P y(t) - 2y^T(t-h_1) W_{0j} f(\cdot) \\
& - 2y^T(t-h_1) W_{1j} g(\cdot) + y^T(t-h_1) B_j B_j^T y(t) = 0, \\
& 2y^T(t-h(t)) P \dot{y}(t) + 2y^T(t-h(t)) A_j P y(t) - 2y^T(t-h(t)) W_{0j} f(\cdot) \\
& - 2y^T(t-h(t)) W_{1j} g(\cdot) + y^T(t-h(t)) B_j B_j^T y(t) = 0. \quad (3.8)
\end{aligned}$$

Adding all the zero items of (3.8) and $f^0(t, x(t), x(t-h(t)), u(t)) - f^0(t, x(t), x(t-h(t)), u(t)) = 0$, respectively, into (3.7) and using the condition (2.4) for the following estimations:

$$\begin{aligned}
f^0(t, x(t), x(t-h(t)), u(t)) & \leq \langle Q_1 x(t), x(t) \rangle + \langle Q_2 x(t-h(t)), x(t-h(t)) \rangle \\
& + \langle R u(t), u(t) \rangle \\
& = \langle P Q_1 P y(t), y(t) \rangle + \langle P Q_2 P y(t-h(t)), y(t-h(t)) \rangle \\
& + 0.25 \langle B_j R B_j^T y(t), y(t) \rangle, \\
2 \langle W_{0j} f(x), y \rangle & \leq \langle W_{0j} D_0 W_{0j}^T y, y \rangle + \langle D_0^{-1} f(x), f(x) \rangle, \\
2 \langle W_{1j} g(z), y \rangle & \leq \langle W_{1j} D_1 W_{1j}^T y, y \rangle + \langle D_1^{-1} g(z), g(z) \rangle, \\
2 \langle D_0^{-1} f(x), f(x) \rangle & \leq \langle F D_0^{-1} F x, x \rangle, \\
2 \langle D_1^{-1} g(z), g(z) \rangle & \leq \langle E D_1^{-1} E z, z \rangle,
\end{aligned}$$

we obtain

$$\begin{aligned} \dot{V}(\cdot) + 2\alpha V(\cdot) \leq & \zeta^T(t) \mathcal{E}_j \zeta(t) + y^T(t) S_{1j} y(t) + y^T(t-h(t)) S_{2j} y(t-h(t)) \\ & - f^0(t, x(t), x(t-h(t)), u(t)), \end{aligned} \quad (3.9)$$

where $\zeta(t) = [y(t), \dot{y}(t), y(t-h_0), y(t-h_1), y(t-h(t))]$, and

$$\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix},$$

$$S_{1j} = -PA_j - A_j^T P - \sum_{i=0}^1 e^{-2\alpha h_i} H_i + 4PFD_0^{-1}FP + PQ_1P,$$

$$S_{2j} = W_{1j}D_1W_{1j}^T - e^{-2\alpha h_2}U + 4PED_1^{-1}EP + PQ_2P.$$

Note that by the Schur complement lemma, Proposition 2.1, the conditions $S_{1j} < 0$ and $S_{2j} < 0$ are equivalent to the conditions (3.2) and (3.3), respectively. Therefore, by conditions (3.1), (3.2), (3.3), we obtain from (3.9) that

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \geq 0. \quad (3.10)$$

Integrating both sides of (3.10) from 0 to t , we obtain

$$V(t, x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \geq 0.$$

Furthermore, taking condition (3.5) into account, we have

$$\lambda_1 \|x(t, \phi)\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \|\phi\|^2,$$

then

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad t \geq 0,$$

which concludes the exponential stability of closed-loop system (2.5). To prove the optimal level of the cost function (2.3), we derive from (3.9) and (3.1)-(3.3) that

$$\dot{V}(t, z_t) \leq -f^0(t, x(t), x(t-h(t)), u(t)), \quad t \geq 0. \quad (3.11)$$

Integrating both sides of (3.11) from 0 to t leads to

$$\int_0^t f^0(t, x(t), x(t-h(t)), u(t)) dt \leq V(0, z_0) - V(t, z_t) \leq V(0, z_0),$$

due to $V(t, z_t) \geq 0$. Hence, letting $t \rightarrow +\infty$, we have

$$J = \int_0^\infty f^0(t, x(t), x(t-h(t)), u(t)) dt \leq V(0, z_0) \leq \lambda_2 \|\phi\|^2 = J^*.$$

This completes the proof of the theorem. $\square$

Example 3.1 Consider the switched recurrent neural networks with interval time-varying delays (2.1), where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, & A_2 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, & W_{01} &= \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.8 \end{bmatrix}, \\ W_{02} &= \begin{bmatrix} -0.7 & 0.3 \\ 0.4 & -0.9 \end{bmatrix}, & W_{11} &= \begin{bmatrix} -0.4 & 0.2 \\ 0.3 & -0.3 \end{bmatrix}, & W_{12} &= \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.4 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, & E &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, & F &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix}, \\ Q_1 &= \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.7 \end{bmatrix}, & Q_2 &= \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}, & R &= \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.9 \end{bmatrix}, \\ \begin{cases} h(t) = 0.1 + 1.2652 \sin^2 t & \text{if } t \in \mathcal{I} = \bigcup_{k \geq 0} [2k\pi, (2k+1)\pi], \\ h(t) = 0 & \text{if } t \in \mathbb{R}^+ \setminus \mathcal{I}. \end{cases} \end{aligned}$$

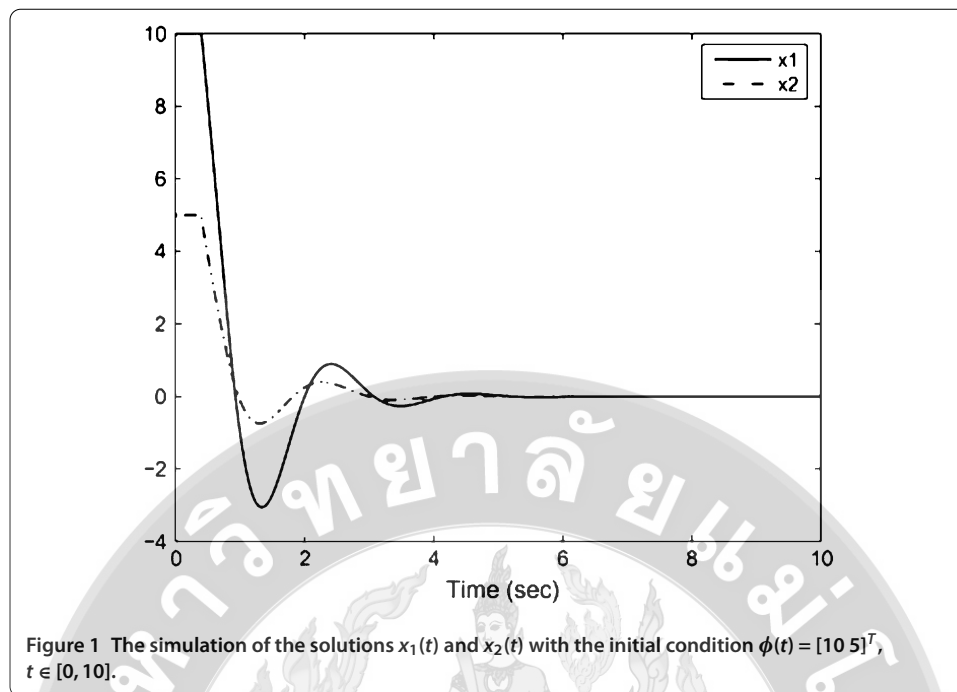
Note that $h(t)$ is non-differentiable, therefore, the stability criteria proposed in [4–7, 9–13, 15–18, 21–24] are not applicable to this system. Given $\alpha = 0.3$, $h_0 = 0.1$, $h_1 = 1.3652$, by using the Matlab LMI toolbox, we can solve for P , U , G_0 , G_1 , H_0 , H_1 , D_0 , and D_1 which satisfy the conditions (3.1)–(3.3) in Theorem 3.1.

A set of solutions is as follows:

$$\begin{aligned} P &= \begin{bmatrix} 1.5219 & -0.3659 \\ -0.3659 & 2.2398 \end{bmatrix}, & U &= \begin{bmatrix} 3.1239 & -0.2365 \\ -0.2365 & 3.0123 \end{bmatrix}, \\ G_0 &= \begin{bmatrix} 1.3225 & 0.0258 \\ 0.0258 & 1.2698 \end{bmatrix}, & G_1 &= \begin{bmatrix} 2.2368 & 0.0148 \\ 0.0148 & 3.1121 \end{bmatrix}, \\ H_0 &= \begin{bmatrix} 2.2189 & 0.1238 \\ 0.1238 & 1.2368 \end{bmatrix}, & H_1 &= \begin{bmatrix} 2.3225 & 0.0369 \\ 0.0369 & 2.1897 \end{bmatrix}, \\ D_0 &= \begin{bmatrix} 2.9870 & 0 \\ 0 & 3.2589 \end{bmatrix}, & D_1 &= \begin{bmatrix} 3.2698 & 0 \\ 0 & 4.3258 \end{bmatrix}. \end{aligned}$$

Then

$$\begin{aligned} u_1(t) &= 0.2579x_1(t) + 0.2589x_2(t), \quad t \geq 0, \\ u_2(t) &= 0.1397x_1(t) + 0.2176x_2(t), \quad t \geq 0, \end{aligned}$$



are a guaranteed cost control law and the cost given by

$$J^* = 1.1268 \|\phi\|^2.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 2.3257 e^{-0.3t} \|\phi\|, \quad \forall t \geq 0.$$

The trajectories of solution of switched recurrent neural networks is shown in Figure 1, respectively.

4 Conclusions

In this paper, the problem of guaranteed cost control for Hopfield neural networks with interval non-differentiable time-varying delay has been studied. A nonlinear quadratic cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of time-varying Lyapunov-Krasovskii functionals, a switching rule for the exponential stabilization for the system is designed via linear matrix inequalities. A memoryless state feedback guaranteed cost controller design has been presented and sufficient conditions for the existence of the guaranteed cost state-feedback for the system have been derived in terms of LMIs.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

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DELAY-DEPENDENT ASYMPTOTICAL STABILIZATION CRITERION OF RECURRENT NEURAL NETWORKS

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Keywords: Neural networks; Time-varying Delay; Stability; Quadratic Lyapunov functional approach.

Abstract. This paper deals with the problem of delay-dependent stability criterion of discrete-time recurrent neural networks with time-varying delays. Based on quadratic Lyapunov functional approach and free-weighting matrix approach, some linear matrix inequality criteria are found to guarantee delay-dependent asymptotical stability of these systems. And one example illustrates the exactness of the proposed criteria.

Introduction

A recurrent neural network (RNNs) is a very important tool for many application areas such as associative memory, pattern recognition, signal processing, model identification and combinatorial optimization. With the development of research on RNNs in theory and application, the model is more and more complex. Parameter uncertainties and nonautonomous phenomena often exist in real systems due to modeling inaccuracies [1, 2]. Particularly when we consider a longterm dynamical behavior of the system and consider seasonality of the changing environment, the parameters of the system usually will change with time [3, 4]. Simultaneously, in implementations of artificial neural networks, time delay may occur due to finite switching speeds of the amplifiers and communication time [5, 6]. In order to model those systems with neural networks, the neural networks with time-varying delay appear in many papers [7, 8]. So in this paper we consider the stability of the following discrete-time recurrent neural networks:

In this paper, we consider control discrete-time system of neural networks of the form

$$v(k+1) = Cv(k) + AS(v(k)) + BS(v(k-h(k))) + Du(k) + f, \quad (1)$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $0 < h_1 \leq h(k) \leq h_2, \forall k = 0, 1, 2, \dots$, $C = \text{diag}\{c_1, \dots, c_n\}$, $c_i \geq 0, i = 1, 2, \dots, n$ is the $n \times n$ constant relaxation matrix, A, B are the $n \times n$ constant weight matrix, D is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector, $f = (f_1, \dots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1, 1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, \dots, n$.

The asymptotic stability of the zero solution of the delay-differential system of Hopfield neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the control discrete-time system of neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

Preliminaries

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all non-negative real numbers; $\mathbb{Z}^+$ denotes the set of all non-negative integers; $\mathbb{R}^n$ denotes the n -finite-dimensional Euclidean space with the Euclidean norm $\|\cdot\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbb{R}^{n \times m}$ denotes the set of all $(n \times m)$ -matrices; and A^T denotes the transpose of the matrix A ;

Matrix $Q \in \mathbb{R}^{n \times n}$ is positive semidefinite ($Q \geq 0$) if $x^T Q x \geq 0$, for all $x \in \mathbb{R}^n$. If $x^T Q x > 0$ ($x^T Q x < 0$, resp.) for any $x \neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0$, ($Q < 0$, resp.). It is easy to verify that $Q > 0$, ($Q < 0$, resp.) iff $\exists \beta > 0$: $x^T Q x \geq \beta \|x\|^2$, $\forall x \in \mathbb{R}^n$, ($\exists \beta > 0$: $x^T Q x \leq -\beta \|x\|^2$, $\forall x \in \mathbb{R}^n$, resp.).

Lemma 1. [1] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x): \mathbb{R}^n \rightarrow \mathbb{R}^+$ such that

$$\exists \beta > 0: \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\beta \|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is asymptotically stable.

Main Results

In this section, we consider the sufficient condition for asymptotic stability of the zero solution v^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0$, $S(0) = 0$ and $f = 0$ (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*)$). The new form of (1) is now given by

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k). \quad (2)$$

This is a basic requirement for controller design. Now, we are interested designing a feedback controller for the system (2) as $u(k) = Kx(k)$, where K is $n \times m$ constant control gain matrix.

The new form of (2) is now given by

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + DKx(k). \quad (3)$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i = 1, 2, \dots, n$ is bounded and monotonically nondecreasing on $\mathbb{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i > 0$, $i = 1, 2, \dots, n$ such that

$$|s_i(r_1) - s_i(r_2)| \leq l_i |r_1 - r_2|, \quad \forall r_1, r_2 \in \mathbb{R}. \quad (4)$$

By condition (4), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \leq l_i |x_i|, \quad i = 1, 2, \dots, n. \quad (5)$$

Theorem 1. The zero solution of the control discrete-time system of neural networks (3) is asymptotically stable if there exist symmetric positive definite matrices P, G, W, R satisfying the following matrix inequalities of the form

$$\Psi = \begin{pmatrix} (1,1) & 0 \\ 0 & (2,2) \end{pmatrix} < 0, \quad (6)$$

where

$$(1,1) = CPC + CPDK + K^T D^T PC + K^T D^T PDK - P + CPAL + L^T A^T PC$$

$$+ K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W,$$

$$(1,2) = CPBL + K^T D^T PBL + L^T A^T PBL, \quad (2,1) = L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL,$$

$$(2,2) = L^T B^T PBL - G - \hat{h}R, \quad \text{and } \hat{h} = h_2 - h_1 + 1.$$

Proof. Consider the Lyapunov function candidate, where

$$V_1(x(k)) = x^T(k)Px(k),$$

$$V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i),$$

$$V_3(x(k)) = \sum_{j=k-h_2+1}^{k-h_1} \sum_{i=j}^{k-1} x^T(i) W x(i),$$

$$V_4(x(k)) = \sum_{i=k-h(k)}^{k-1} (h(k) - k + i) x^T(i) R x(i).$$

The Lyapunov difference of the system along trajectory of solution of (3) is given by $\Delta V_1(x(k)) = V_1(x(k+1)) - V_1(x(k))$

$$\begin{aligned} &= [Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k)]^T \\ &\quad \times P [Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k)] \\ &\quad - x^T(k) P x(k) \\ &= x^T(k) [CPC + CPDK + K^T D^T PC + K^T D^T PDK - P] x(k) \\ &\quad + x^T(k) CPAS(x(k)) + S^T(x(k)) A^T PC x(k) \\ &\quad + x^T(k) CPBS(x(k-h(k))) + S^T(x(k-h(k))) B^T PC x(k). \end{aligned}$$

As a result, we obtain

$$\begin{aligned} \Delta V &\leq x^T(k) [CPC + CPDK + K^T D^T PC + K^T D^T PDK - P \\ &\quad + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h} W] x(k) \\ &\quad + x^T(k) [CPBL + K^T D^T PBL + L^T A^T PBL] x(k-h(k)) \\ &\quad + x^T(k-h(k)) [L^T B^T PC + L^T B^T PDK x(k) + L^T B^T PAL] x(k) \\ &\quad + x^T(k-h(k)) [L^T B^T PBL - G - \hat{h} R] x(k-h(k)), \\ &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}^T \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix} \\ &= y^T(k) \Psi y(k), \end{aligned}$$

Where,

$$\begin{aligned} (1,1) &= CPC + CPDK + K^T D^T PC + K^T D^T PDK - P \\ &\quad + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h} W, \\ (1,2) &= CPBL + K^T D^T PBL + L^T A^T PBL, \\ (2,1) &= L^T B^T PC + L^T B^T PDK x(k) + L^T B^T PAL, \\ (2,2) &= L^T B^T PBL - G - \hat{h} R, \\ y(k) &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}. \end{aligned}$$

By the condition (6), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from

Lemma 1. This completes the proof.

Remark 1. Theorem 1 gives a sufficient condition for the asymptotic stability of control discrete-time system of neural networks (3) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed. But [9, 10] these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed.

Conclusions

This paper was dedicated to the delay-dependent stability of discrete-time recurrent neural networks with time-varying delay. A less conservative LMI-based globally stability criterion is obtained with quadratic Lyapunov functional approach and free-weighting matrix approach for periodic discrete-time recurrent neural networks with a time-varying delay. One example illustrates the exactness of the proposed criterion.

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Materials Engineering and Automatic Control II

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Delay-Dependent Asymptotical Stabilization Criterion of Recurrent Neural Networks

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New design switching rule for the robust mean square stability of uncertain stochastic discrete-time hybrid systems

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Abstract—This paper is concerned with robust mean square stability of uncertain stochastic switched discrete time-delay systems. The system to be considered is subject to interval time-varying delays, which allows the delay to be a fast time-varying function and the lower bound is not restricted to zero. Based on the discrete Lyapunov functional, a switching rule for the robust mean square stability for the uncertain stochastic discrete time-delay system is designed via linear matrix inequalities.

I. INTRODUCTION

Switched systems constitute an important class of hybrid systems. Such systems can be described by a family of continuous-time subsystems (or discrete-time subsystems) and a rule that orchestrates the switching between them. It is well known that a wide class of physical systems in power systems, chemical process control systems, navigation systems, automobile speed change system, and so forth may be appropriately described by the switched model [1-7]. In the study of switched systems, most works have been centralized on the problem of stability. In the last two decades, there has been increasing interest in the stability analysis for such switched systems; see, for example, [8, 9] and the references cited therein. Two important methods are used to construct the switching law for the stability analysis of the switched systems. One is the state-driven switching strategy [9]; the other is the time-driven switching strategy [8]. A switched system is a hybrid dynamical system consisting of a finite number of subsystems and a logical rule that manages switching between these subsystems (see, e.g., [1-10] and the references therein).

The main approach for stability analysis relies on the use of Lyapunov-Krasovskii functional and linear matrix inequality (LMI) approach for constructing a common Lyapunov function [11, 12, 13]. Although many important results have been obtained for switched linear continuous-time systems, there are few results concerning the stability of switched linear discrete systems with time-varying delays. In [14, 15], a class of switching signals has been identified for the considered switched discrete-time delay systems to be stable under the average dwell time scheme.

This paper studies robust mean square stability problem for uncertain stochastic switched linear discrete-time delay

with interval time-varying delays. Specifically, our goal is to develop a constructive way to design switching rule to robustly mean square stable the uncertain stochastic linear discrete-time delay systems. By using improved Lyapunov-Krasovskii functional combined with LMIs technique, we propose new criteria for the robust mean square stability of the uncertain stochastic linear discrete-time delay system. Compared to the existing results, our result has its own advantages. First, the time delay is assumed to be a time-varying function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, the delay function is bounded but not restricted to zero. Second, the approach allows us to design the switching rule for robust mean square stability in terms of LMIs.

The paper is organized as follows: Section II presents definitions and some well-known technical propositions needed for the proof of the main results. Switching rule for the robust mean square stability is presented in Section III.

II. PRELIMINARIES

The following notations will be used throughout this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n -dimensional space with the scalar product of two vectors $\langle x, y \rangle$ or $x^T y$; $R^{n \times r}$ denotes the space of all matrices of $(n \times r)$ -dimension. N^+ denotes the set of all non-negative integers; A^T denotes the transpose of A ; a matrix A is symmetric if $A = A^T$.

Matrix A is semi-positive definite ($A \geq 0$) if $\langle Ax, x \rangle \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $\langle Ax, x \rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. $\lambda(A)$ denotes the set of all eigenvalues of A ; $\lambda_{\min}(A) = \min\{Re\lambda : \lambda \in \lambda(A)\}$.

Consider a uncertain stochastic discrete systems with interval time-varying delay of the form

$$\begin{aligned} x(k+1) &= (A_\gamma + \Delta A_\gamma(k))x(k) + (B_\gamma + \Delta B_\gamma(k))x(k-d(k)) \\ &\quad + \sigma_\gamma(x(k), x(k-d(k)), k)\omega(k), \\ k &\in N^+, \quad x(k) = v_k, \quad k = -d_2, -d_2 + 1, \dots, 0, \end{aligned} \quad (1)$$

where $x(k) \in R^n$ is the state, $\gamma(\cdot) : R^n \rightarrow \mathcal{N} := \{1, 2, \dots, N\}$ is the switching rule, which is a function

depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(k)) = i$ implies that the system realization is chosen as the i^{th} system, $i = 1, 2, \dots, N$. It is seen that the system (1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(k)$ hits predefined boundaries. $A_i, B_i, i = 1, 2, \dots, N$ are given constant matrices and the time-varying uncertain matrices $\Delta A_i(k)$ and $\Delta B_i(k)$ are defined by: $\Delta A_i(k) = E_{ia}F_{ia}(k)H_{ia}$, $\Delta B_i(k) = E_{ib}F_{ib}(k)H_{ib}$, where $E_{ia}, E_{ib}, H_{ia}, H_{ib}$ are known constant real matrices with appropriate dimensions. $F_{ia}(k), F_{ib}(k)$ are unknown uncertain matrices satisfying

$$F_{ia}^T(k)F_{ia}(k) \leq I, \quad F_{ib}^T(k)F_{ib}(k) \leq I, \quad k = 0, 1, 2, \dots, \quad (2)$$

where I is the identity matrix of appropriate dimension, $\omega(k)$ is a scalar Wiener process (Brownian Motion) on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$E[\omega(k)] = 0, \quad E[\omega^2(k)] = 1, \quad E[\omega(i)\omega(j)] = 0 (i \neq j), \quad (3)$$

and $\sigma_i: R^n \times R^n \times R \rightarrow R^n, i = 1, 2, \dots, N$ is the continuous function, and is assumed to satisfy that

$$\begin{aligned} \sigma_i^T(x(k), x(k-d(k)), k) \sigma_i(x(k), x(k-d(k)), k) &\leq \\ \rho_{i1}x^T(k)x(k) + \rho_{i2}x^T(k-d(k))x(k-d(k)), & \quad (4) \\ x(k), x(k-d(k)) \in R^n, & \end{aligned}$$

where $\rho_{i1} > 0$ and $\rho_{i2} > 0, i = 1, 2, \dots, N$ are known constant scalars. The time-varying function $d(k): N^+ \rightarrow N^+$ satisfies the following condition:

$$0 < d_1 \leq d(k) \leq d_2, \quad \forall k \in N^+$$

Remark 2.1. It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 2.1. The uncertain stochastic switched system (1) is robustly stable if there exists a switching function $\gamma(\cdot)$ such that the zero solution of the uncertain stochastic switched system is robustly stable.

Definition 2.2. The system of matrices $\{J_i\}, i = 1, 2, \dots, N$, is said to be strictly complete if for every $x \in R^n \setminus \{0\}$ there is $i \in \{1, 2, \dots, N\}$ such that $x^T J_i x < 0$.

It is easy to see that the system $\{J_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^N \alpha_i = R^n \setminus \{0\},$$

where

$$\alpha_i = \{x \in R^n : x^T J_i x < 0\}, i = 1, 2, \dots, N.$$

Definition 2.3. The discrete-time system (1) is robustly stable

in the mean square if there exists a positive definite scalar function $V(k, x(k)): R^n \times R^n \rightarrow R$ such that

$$E[\Delta V(k, x(k))] = E[V(k+1, x(k+1)) - V(k, x(k))] < 0, \text{ along any trajectory of solution of the system (1).}$$

Proposition 2.1. [16] The system $\{J_i\}, i = 1, 2, \dots, N$, is strictly complete if there exist $\delta_i \geq 0, i = 1, 2, \dots, N, \sum_{i=1}^N \delta_i > 0$ such that

$$\sum_{i=1}^N \delta_i J_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Proposition 2.2. (Cauchy inequality) For any symmetric positive definite matrix $N \in M^{n \times n}$ and $a, b \in R^n$ we have

$$\pm a^T b \leq a^T N a + b^T N^{-1} b.$$

Proposition 2.3. [16] Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \leq I$. For any $\epsilon > 0$, we have

$$EFH + H^T F^T E^T \leq \epsilon E E^T + \epsilon^{-1} H^T H.$$

III. MAIN RESULTS

Let us set

$$W_i = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

where

$$W_{i11} = Q - P,$$

$$W_{i12} = S_1 - S_1 A_i,$$

$$W_{i13} = -S_1 B_i,$$

$$W_{i22} = P + S_1 + S_1^T + H_{ia}^T H_{ia} + S_1 E_{ib} E_{ib}^T S_1^T,$$

$$W_{i23} = -S_1 B_i,$$

$$W_{i33} = -Q + 2H_{ib}^T H_{ib} + 2\rho_{i2}I,$$

$$\begin{aligned} J_i &= (d_2 - d_1)Q - S_1 A_i - A_i^T S_1^T + 2S_1 E_{ia} E_{ia}^T S_1^T \\ &\quad + S_1 E_{ib} E_{ib}^T S_1^T + H_{ia}^T H_{ia} + 2\rho_{i1}I, \end{aligned}$$

$$\alpha_i = \{x \in R^n : x^T J_i x < 0\}, i = 1, 2, \dots, N, \quad (5)$$

$$\bar{\alpha}_1 = \alpha_1, \quad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2, 3, \dots, N.$$

The main result of this paper is summarized in the following theorem.

Theorem 1. The uncertain stochastic switched system (1) is robustly stable in the mean square if there exist symmetric positive definite matrices $P > 0, Q > 0$ and matrix S_1 satisfying the following conditions

(i) $\exists \delta_i \geq 0, i = 1, 2, \dots, N, \sum_{i=1}^N \delta_i > 0 : \sum_{i=1}^N \delta_i J_i < 0.$

(ii) $W_i < 0, i = 1, 2, \dots, N.$

The switching rule is chosen as $\gamma(x(k)) = i$, whenever $x(k) \in \bar{\alpha}_i$.

Proof. Consider the following Lyapunov-Krasovskii functional for any i th system (1)

$$V(k) = V_1(k) + V_2(k) + V_3(k),$$

where

$$V_1(k) = x^T(k)Px(k), \quad V_2(k) = \sum_{i=k-d(k)}^{k-1} x^T(i)Qx(i),$$

$$V_3(k) = \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j+1}^{k-1} x^T(l)Qx(l),$$

We can verify that

$$\lambda_1 \|x(k)\|^2 \leq V(k). \quad (6)$$

Let us set $\xi(k) = [x(k) \ x(k+1) \ x(k-d(k)) \ \omega(k)]^T$, and

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} P & 0 & 0 & 0 \\ I & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}.$$

Then, the difference of $V_1(k)$ along the solution of the system (1) and taking the mathematical expectation, we obtained

$$E[\Delta V_1(k)] = E[x^T(k+1)Px(k+1) - x^T(k)Px(k)]$$

$$= E[\xi^T(k)H\xi(k) - 2\xi^T(k)G^T \begin{pmatrix} 0.5x(k) \\ 0 \\ 0 \\ 0 \end{pmatrix}]. \quad (7)$$

because of

$$\xi^T(k)H\xi(k) = x(k+1)Px(k+1),$$

$$2\xi^T(k)G^T \begin{pmatrix} 0.5x(k) \\ 0 \\ 0 \\ 0 \end{pmatrix} = x^T(k)Px(k).$$

Using the expression of system (1)

$$0 = -S_1x(k+1) + S_1(A_i + E_{ia}F_{ia}(k)H_{ia})x(k)$$

$$+ S_1(B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + S_1\sigma_i\omega(k),$$

$$0 = -\sigma_i^T x(k+1) + \sigma_i^T (A_i + E_{ia}F_{ia}(k)H_{ia})x(k)$$

$$+ \sigma_i^T (B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + \sigma_i^T \sigma_i \omega(k),$$

we have

$$E[-2\xi^T(k)G^T \begin{pmatrix} 0.5x(k) \\ [-S_1x(k+1) + S_1(A_i + E_{ia}F_{ia}(k)H_{ia})x(k) \\ + S_1(B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + S_1\sigma_i\omega(k)] \\ 0 \\ [-\sigma_i^T x(k+1) + \sigma_i^T (A_i + E_{ia}F_{ia}(k)H_{ia})x(k) \\ + \sigma_i^T (B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + \sigma_i^T \sigma_i \omega(k)] \end{pmatrix}]$$

Therefore, from (7) it follows that

$$E[\Delta V_1(k)] = E[x^T(k)[-P - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia}$$

$$- A_i^T S_1^T - H_{ia}^T F_{ia}^T(k)E_{ia}S_1^T]x(k)$$

$$+ 2x^T(k)[S_1 - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia}]x(k+1)$$

$$+ 2x^T(k)[-S_1B_i - S_1E_{ib}F_{ib}(k)H_{ib}]x(k-d(k))$$

$$+ 2x^T(k)[-S_1\sigma_i - \sigma_i^T A_i - \sigma_i^T E_{ia}F_{ia}(k)H_{ia}]\omega(k)$$

$$+ x(k+1)[S_1 + S_1^T]x(k+1)$$

$$+ 2x(k+1)[-S_1B_i - S_1(E_{ib}F_{ib}(k)H_{ib})]x(k-d(k))$$

$$+ 2x(k+1)[\sigma_i^T - S_1\sigma_i]\omega(k)$$

$$+ x^T(k-d(k))[-\sigma_i^T B_i - \sigma_i^T E_{ib}F_{ib}(k)H_{ib}]\omega(k)$$

$$+ \omega^T(k)[-2\sigma_i^T \sigma_i]\omega(k)],$$

By assumption (3), we have

$$E[\Delta V_1(k)] = E[x^T(k)[-P - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia}$$

$$- A_i^T S_1^T - H_{ia}^T F_{ia}^T(k)E_{ia}S_1^T]x(k)$$

$$+ 2x^T(k)[S_1 - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia}]x(k+1)$$

$$+ 2x^T(k)[-S_1B_i - S_1E_{ib}F_{ib}(k)H_{ib}]x(k-d(k))$$

$$+ x(k+1)[S_1 + S_1^T]x(k+1)$$

$$+ 2x(k+1)[-S_1B_i - S_1E_{ib}F_{ib}(k)H_{ib}]x(k-d(k))$$

$$- 2\sigma_i^T \sigma_i],$$

Applying Proposition 2.2, Proposition 2.3, condition (2) and assumption (4), the following estimations hold

$$-S_1E_{ia}F_{ia}(k)H_{ia} - H_{ia}^T F_{ia}^T(k)E_{ia}^T S_1^T \leq S_1E_{ia}E_{ia}^T S_1^T + H_{ia}^T H_{ia},$$

$$-2x^T(k)S_1E_{ia}F_{ia}(k)H_{ia}x(k+1) \leq$$

$$x^T(k)S_1E_{ia}E_{ia}^T S_1^T x(k) + x(k+1)^T H_{ia}^T H_{ia}x(k+1),$$

$$-2x^T(k)S_1E_{ib}F_{ib}(k)H_{ib}x(k-d(k)) \leq$$

$$x^T(k)S_1E_{ib}E_{ib}^T S_1^T x(k) + x(k-d(k))^T H_{ib}^T H_{ib}x(k-d(k)),$$

$$-2x^T(k+1)S_1E_{ib}F_{ib}(k)H_{ib}x(k-d(k)) \leq$$

$$x^T(k+1)S_1E_{ib}E_{ib}^T S_1^T x(k+1) + x(k-d(k))^T H_{ib}^T H_{ib}x(k-d(k)),$$

$$-\sigma_i^T(x(k), x(k-d(k)), k)\sigma_i(x(k), x(k-d(k)), k) \leq$$

$$\rho_{i1}x^T(k)x(k) + \rho_{i2}x^T(k-d(k))x(k-d(k)).$$

Therefore, we have

$$\begin{aligned} E[\Delta V_1(k)] &= E[x^T(k)[-P - S_1A_i - A_i^T S_1^T \\ &\quad + 2S_1E_{ia}E_{ia}^T S_1^T \\ &\quad + S_1E_{ib}E_{ib}^T S_1^T + S_2E_{ia}E_{ia}^T S_2^T \\ &\quad + H_{ia}^T H_{ia} + 2\rho_{i1}I]x(k) \\ &\quad + 2x^T(k)[S_1 - S_1A_i]x(k+1) \\ &\quad + 2x^T(k)[-S_1B_i - S_2A_i]x(k-d(k)) \quad (8) \\ &\quad + x(k+1)[S_1 + S_1^T + H_{ia}^T H_{ia} \\ &\quad + S_1E_{ib}E_{ib}^T S_1^T]x(k+1) \\ &\quad + 2x(k+1)[S_2 - S_1B_i]x(k-d(k)) \\ &\quad + x^T(k-d(k))[2H_{ib}^T H_{ib} \\ &\quad + 2\rho_{i2}I]x(k-d(k))], \end{aligned}$$

The difference of $V_2(k)$ is given by

$$\begin{aligned} E[\Delta V_2(k)] &= E\left[\sum_{i=k+1-d(k+1)}^k x^T(i)Qx(i) \right. \\ &\quad \left. - \sum_{i=k-d(k)}^{k-1} x^T(i)Qx(i)\right] \\ &= E\left[\sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) \right. \\ &\quad \left. + x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k)) \right. \\ &\quad \left. + \sum_{i=k+1-d_1}^{k-1} x^T(i)Qx(i) \right. \\ &\quad \left. - \sum_{i=k+1-d(k)}^{k-1} x^T(i)Qx(i)\right]. \quad (9) \end{aligned}$$

Since $d(k) \geq d_1$ we have

$$\sum_{i=k+1-d_1}^{k-1} x^T(i)Qx(i) - \sum_{i=k+1-d(k)}^{k-1} x^T(i)Qx(i) \leq 0,$$

and hence from (9) we have

$$\begin{aligned} E[\Delta V_2(k)] &\leq E\left[\sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) \right. \\ &\quad \left. + x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k))\right]. \quad (10) \end{aligned}$$

The difference of $V_3(k)$ is given by

$$\begin{aligned} E[\Delta V_3(k)] &= E\left[\sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j}^k x^T(l)Qx(l) \right. \\ &\quad \left. - \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j+1}^{k-1} x^T(l)Qx(l)\right] \\ &= E\left[\sum_{j=-d_2+2}^{-d_1+1} \left[\sum_{l=k+j}^{k-1} x^T(l)Qx(l) + x^T(k)Q(\xi)x(k) \right. \right. \\ &\quad \left. \left. - \sum_{l=k+j}^{k-1} x^T(l)Qx(l) \right. \right. \\ &\quad \left. \left. - x^T(k+j-1)Qx(k+j-1)\right]\right] \\ &= E\left[\sum_{j=-d_2+2}^{-d_1+1} \left[x^T(k)Qx(k) \right. \right. \\ &\quad \left. \left. - x^T(k+j-1)Qx(k+j-1)\right]\right] \\ &= E[(d_2 - d_1)x^T(k)Qx(k) \\ &\quad - \sum_{j=k+1-d_2}^{k-d_1} x^T(j)Qx(j)]. \quad (11) \end{aligned}$$

Since $d(k) \leq d_2$, and

$$\sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) - \sum_{i=k+1-d_2}^{k-d_1} x^T(i)Qx(i) \leq 0,$$

we obtain from (10) and (11) that

$$E[\Delta V_2(k) + \Delta V_3(k)] \leq E[(d_2 - d_1 + 1)x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k))]. \quad (12)$$

Therefore, combining the inequalities (8), (12) gives

$$E[\Delta V(k)] \leq E[x^T(k)J_i x(k) + \psi^T(k)W_i \psi(k)], \quad (13)$$

where

$$\psi(k) = [x(k) \ x(k+1) \ x(k-d(k))]^T,$$

$$W_i = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

$$W_{i11} = Q - P,$$

$$W_{i12} = S_1 - S_1A_i,$$

$$W_{i13} = -S_1B_i,$$

$$W_{i22} = P + S_1 + S_1^T + H_{ia}^T H_{ia} + S_1E_{ib}E_{ib}^T S_1^T,$$

$$W_{i23} = -S_1B_i,$$

$$W_{i33} = -Q + 2H_{ib}^T H_{ib} + 2\rho_{i2}I,$$

and

$$\begin{aligned} J_i &= (d_2 - d_1)Q - S_1A_i - A_i^T S_1^T + 2S_1E_{ia}E_{ia}^T S_1^T + S_1E_{ib}E_{ib}^T S_1^T \\ &\quad + H_{ia}^T H_{ia} + 2\rho_{i1}I. \end{aligned}$$

Therefore, we finally obtain from (13) and the condition (ii) that

$$E[\Delta V(k)] < E[x^T(k)J_i x(k)], \quad \forall i = 1, 2, \dots, N, \\ k = 0, 1, 2, \dots$$

We now apply the condition (i) and Proposition 2.1., the system J_i is strictly complete, and the sets α_i and $\bar{\alpha}_i$ by (5) are well defined such that

$$\bigcup_{i=1}^N \alpha_i = R^n \setminus \{0\}, \\ \bigcup_{i=1}^N \bar{\alpha}_i = R^n \setminus \{0\}, \quad \bar{\alpha}_i \cap \bar{\alpha}_j = \emptyset, i \neq j.$$

Therefore, for any $x(k) \in R^n, k = 1, 2, \dots$, there exists $i \in \{1, 2, \dots, N\}$ such that $x(k) \in \bar{\alpha}_i$. By choosing switching rule as $\gamma(x(k)) = i$ whenever $x(k) \in \bar{\alpha}_i$, from the condition (13) we have

$$E[\Delta V(k)] \leq E[x^T(k)J_i x(k)] < 0, \quad k = 1, 2, \dots,$$

which, combining the condition (6), Definition 2.3 and the Lyapunov stability theorem [16], concludes the proof of the theorem in the mean square.

IV. CONCLUSION

This paper has proposed a switching design for the robust stability of uncertain stochastic switched discrete time-delay systems with interval time-varying delays. Based on the discrete Lyapunov functional, a switching rule for the robust stability for the uncertain stochastic switched discrete time-delay system is designed via linear matrix inequalities.

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New delay-dependent sufficient conditions for the exponential stability of linear hybrid systems with interval time-varying delays

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Abstract—This paper is concerned with exponential stability of switched linear systems with interval time-varying delays. The time delay is any continuous function belonging to a given interval, in which the lower bound of delay is not restricted to zero. By constructing a suitable augmented Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, a switching rule for the exponential stability of switched linear systems with interval time-varying delays and new delay-dependent sufficient conditions for the exponential stability of the systems are first established in terms of LMIs.

I. INTRODUCTION

Switched time-delay systems have been attracting considerable attention during the recent years [1-10], due to the significance both in theory development and practical applications. However, it is worth noting that only the state time delay is considered, and the time delay in the state derivatives is largely ignored in the existing literature. If each subsystem of a switched system has time delay in the state derivatives, then the switched system is called switched neutral system [10-14]. Switched neutral systems exist widely in engineering and social systems, many physical plants can be modelled as switched neutral systems, such as distributed networks and heat exchanges. For example, in [11-16], a switched neutral type delay equation with nonlinear perturbations was exploited to model the drilling system. Unlike other systems, the neutral has time-delay in both the state and derivative. However, it is well-known that time-delay in the system may be a source of instability or bad system performance. Thus many researchers try to study them to find stability criteria for such system with time-delay to be stable. Most of the known results on this problem are derived assuming only that the time-varying delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative: $\dot{h}(t) \leq \delta < 1$. This paper gives the improved results for the exponential stability of switched linear systems with interval time-varying delay. The time delay is assumed to be a time-varying continuous function belonging to a given interval, but not necessary to be differentiable. Specifically, our goal is to develop a constructive way to design switching rule to the exponential stability

of switched linear systems with interval time-varying delay. By constructing argument Lyapunov functional combined with LMI technique, we propose new criteria for the exponential stability of the switched linear system. The delay-dependent stability conditions are formulated in terms of LMIs.

The paper is organized as follows: Section II presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent exponential stability conditions of the switched linear system are presented in Section III.

II. PRELIMINARIES

The following notations will be used in this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n -dimensional space with the scalar product $\langle \cdot, \cdot \rangle$ and the vector norm $\| \cdot \|$; $M^{n \times r}$ denotes the space of all matrices of $(n \times r)$ -dimensions; A^T denotes the transpose of matrix A ; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A ; $\lambda_{\min/\max}(A) = \min/\max\{\operatorname{Re} \lambda; \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\|x_t\| = \sup_{s \in [-h, 0]} \|x(t+s)\|$; $C([0, t], R^n)$ denotes the set of all R^n -valued continuous functions on $[0, t]$; Matrix A is called semi-positive definite ($A \geq 0$) if $\langle Ax, x \rangle \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $\langle Ax, x \rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. $*$ denotes the symmetric term in a matrix.

Consider a linear system with interval time-varying delay of the form

$$\begin{aligned} \dot{x}(t) &= A_\gamma x(t) + D_\gamma x(t - h(t)), \quad t \in R^+, \\ x(t) &= \phi(t), \quad t \in [-h_2, 0], \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state; $\gamma(\cdot) : R^n \rightarrow \mathcal{N} := \{1, 2, \dots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = i$ implies that the system realization is chosen as the i^{th} system, $i = 1, 2, \dots, N$. It is seen that the system (1) can be viewed as

an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries. $A_i, D_i \in M^{n \times n}, i = 1, 2, \dots, N$ are given constant matrices, and $\phi(t) \in C([-h_2, 0], R^n)$ is the initial function with the norm

$\|\phi\| = \sup_{s \in [-h_2, 0]} \|\phi(s)\|$; The time-varying delay function $h(t)$ satisfies

$$0 \leq h_1 \leq h(t) \leq h_2, \quad t \in R^+.$$

The stability problem for switched system (1) is to construct a switching rule that makes the system exponentially stable.

Remark 2.1. It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 2.1. Given $\alpha > 0$. The switched linear system (1) is α -exponentially stable if there exists a switching rule $\gamma(\cdot)$ such that every solution $x(t, \phi)$ of the system satisfies the following condition

$$\exists N > 0: \quad \|x(t, \phi)\| \leq N e^{-\alpha t} \|\phi\|, \quad \forall t \in R^+.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Definition 2.2. The system of matrices $\{J_i\}, i = 1, 2, \dots, N$, is said to be strictly complete if for every $x \in R^n \setminus \{0\}$ there is $i \in \{1, 2, \dots, N\}$ such that $x^T J_i x < 0$.

It is easy to see that the system $\{J_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^N \alpha_i = R^n \setminus \{0\},$$

where

$$\alpha_i = \{x \in R^n: \quad x^T J_i x < 0\}, i = 1, 2, \dots, N.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.1. [17] *The system $\{J_i\}, i = 1, 2, \dots, N$, is strictly complete if there exist $\delta_i \geq 0, i = 1, 2, \dots, N, \sum_{i=1}^N \delta_i > 0$ such that*

$$\sum_{i=1}^N \delta_i J_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Proposition 2.2. (Cauchy inequality) *For any symmetric positive definite matrix $N \in M^{n \times n}$ and $a, b \in R^n$ we have*

$$\pm a^T b \leq a^T N a + b^T N^{-1} b.$$

Proposition 2.3. [18] *For any symmetric positive definite matrix $M \in M^{n \times n}$, scalar $\gamma > 0$ and vector function $\omega: [0, \gamma] \rightarrow R^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_0^\gamma \omega(s) ds \right)^T M \left(\int_0^\gamma \omega(s) ds \right) \leq \gamma \left(\int_0^\gamma \omega^T(s) M \omega(s) ds \right).$$

Proposition 2.4. [19] *Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \leq I$. For any $\epsilon > 0$, we have*

$$EFH + H^T F^T E^T \leq \epsilon E E^T + \epsilon^{-1} H^T H.$$

Proposition 2.5. (Schur complement lemma [20]). *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

III. MAIN RESULTS

Let us set

$$M_i = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix},$$

$$J_i = Q - S_1 A_i - A_i^T S_1^T, \quad (2)$$

$$\alpha_i = \{x \in R^n: \quad x^T J_i x < 0\}, \quad i = 1, 2, \dots, N,$$

$$\bar{\alpha}_1 = \alpha_1, \quad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2, 3, \dots, N,$$

$$\lambda_1 = \lambda_{\min}(P),$$

$$\lambda_2 = \lambda_{\max}(P) + 2h_2 \lambda_{\max}(Q),$$

$$M_{11} = A_i^T P + P A_i + 2\alpha P + Q,$$

$$M_{12} = -S_2 A_i, \quad M_{13} = -S_3 A_i,$$

$$M_{14} = P D_i - S_1 D_i - S_4 A_i, \quad M_{15} = S_1 - S_5 A_i,$$

$$M_{22} = -e^{-2\alpha h_1} Q, \quad M_{24} = -S_2 D_i,$$

$$M_{33} = -e^{-2\alpha h_2} Q, \quad M_{34} = -S_3 D_i,$$

$$M_{44} = -S_4 D_i, \quad M_{45} = S_4 - S_5 D_i,$$

$$M_{55} = S_5 + S_5^T.$$

The main result of this paper is summarized in the following theorem.

we have

Theorem 1. Given $\alpha > 0$. The zero solution of the switched linear system (1) is α -exponentially stable if there exist symmetric positive definite matrices P, Q , and matrices $S_i, i = 1, 2, \dots, 5$ such that satisfying the following conditions

$$(i) \exists \delta_i \geq 0, i = 1, 2, \dots, N, \quad \sum_{i=1}^N \delta_i > 0 : \sum_{i=1}^N \delta_i J_i < 0.$$

$$(ii) \mathcal{M}_i < 0, \quad i = 1, 2, \dots, N.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad \forall t \in \mathbb{R}^+.$$

Proof. We consider the following Lyapunov-Krasovskii functional for the system (1)

$$V(t, x_t) = \sum_{i=1}^3 V_i,$$

where

$$V_1 = x^T(t) P x(t),$$

$$V_2 = \int_{t-h_1}^t e^{2\alpha(s-t)} x^T(s) Q x(s) ds,$$

$$V_3 = \int_{t-h_2}^t e^{2\alpha(s-t)} x^T(s) Q x(s) ds.$$

It easy to check that

$$\lambda_1 \|x(t)\|^2 \leq V(t, x_t) \leq \lambda_2 \|x_t\|^2, \quad \forall t \geq 0, \quad (3)$$

Taking the derivative of V_1 along the solution of system (1) we have

$$\dot{V}_1 = 2x^T(t) P \dot{x}(t)$$

$$= 2x^T(t) [A_i^T P + A_i P] x(t) + 2x^T(t) P D_i x(t-h(t));$$

$$\dot{V}_2 = x^T(t) Q x(t) - e^{-2\alpha h_1} x^T(t-h_1) Q x(t-h_1) - 2\alpha V_2;$$

$$\dot{V}_3 = x^T(t) Q x(t) - e^{-2\alpha h_2} x^T(t-h_2) Q x(t-h_2) - 2\alpha V_3.$$

Therefore, we have

$$\begin{aligned} \dot{V}(\cdot) + 2\alpha V(\cdot) &\leq 2x^T(t) [A_i^T P + A_i P + 2\alpha P + 2Q] x(t) \\ &\quad + 2x^T(t) P D_i x(t-h(t)) \\ &\quad - e^{-2\alpha h_1} x^T(t-h_1) Q x(t-h_1) \\ &\quad - e^{-2\alpha h_2} x^T(t-h_2) Q x(t-h_2). \end{aligned} \quad (4)$$

By using the following identity relation

$$\dot{x}(t) - A_i x(t) - D_i x(t-h(t)) = 0,$$

$$\begin{aligned} &2x^T(t) S_1 \dot{x}(t) - 2x^T(t) S_1 A_i x(t) \\ &\quad - 2x^T(t) S_1 D_i x(t-h(t)) = 0 \\ &2x^T(t-h_1) S_2 \dot{x}(t) - 2x^T(t-h_1) S_2 A_i x(t) \\ &\quad - 2x^T(t-h_1) S_2 D_i x(t-h(t)) = 0 \\ &2x^T(t-h_2) S_3 \dot{x}(t) - 2x^T(t-h_2) S_3 A_i x(t) \\ &\quad - 2x^T(t-h_2) S_3 D_i x(t-h(t)) = 0 \\ &2x^T(t-h(t)) S_4 \dot{x}(t) - 2x^T(t-h(t)) S_4 A_i x(t) \\ &\quad - 2x^T(t-h(t)) S_4 D_i x(t-h(t)) = 0 \\ &2\dot{x}^T(t) S_5 \dot{x}(t) - 2\dot{x}^T(t) S_5 A_i x(t) \\ &\quad - 2\dot{x}^T(t) S_5 D_i x(t-h(t)) = 0 \end{aligned} \quad (5)$$

Adding all the zero items of (5) into (4), we obtain

$$\begin{aligned} \dot{V}(\cdot) + 2\alpha V(\cdot) &\leq x^T(t) [A_i^T P + P A_i + 2\alpha P - S_1 A_i \\ &\quad - A_i^T S_1^T + 2Q] x(t) \\ &\quad + 2x^T(t) [e^{-2\alpha h_1} R - S_2 A_i] x(t-h_1) \\ &\quad + 2x^T(t) [-S_3 A_i] x(t-h_2) + 2x^T(t) [P D_i \\ &\quad - S_1 D_i - S_4 A_i] x(t-h(t)) \\ &\quad + 2x^T(t) [S_1 - S_5 A_i] \dot{x}(t) \\ &\quad + x^T(t-h_1) [-e^{-2\alpha h_1} Q] x(t-h_1) \\ &\quad + 2x^T(t-h_1) [-S_2 D_i] x(t-h(t)) \\ &\quad + 2x^T(t-h_1) S_2 \dot{x}(t) \\ &\quad + x^T(t-h_2) [-e^{-2\alpha h_2} Q] x(t-h_2) \\ &\quad + x^T(t-h_2) [-S_3 D_i] x(t-h(t)) \\ &\quad + 2x^T(t-h_2) S_3 \dot{x}(t) \\ &\quad + x^T(t-h(t)) [-S_4 D_i] x(t-h(t)) \\ &\quad + 2x^T(t-h(t)) [S_4 - S_5 D_i] \dot{x}(t) \\ &\quad + \dot{x}^T(t) [S_5 + S_5^T] \dot{x}(t) \\ &= x^T(t) J_i x(t) + \zeta^T(t) \mathcal{M}_i \zeta(t), \end{aligned} \quad (6)$$

where

$$\zeta(t) = [x(t), x(t-h_1), x(t-h_2), x(t-h(t)), \dot{x}(t)],$$

$$J_i = Q - S_1 A_i - A_i^T S_1^T,$$

$$\mathcal{M}_i = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix},$$

$$\begin{aligned} M_{11} &= A_i^T P + P A_i + 2\alpha P + Q, \\ M_{12} &= -S_2 A_i, \quad M_{13} = -S_3 A_i, \\ M_{14} &= P D_i - S_1 D_i - S_4 A_i, \quad M_{15} = S_1 - S_5 A_i, \\ M_{22} &= -e^{-2\alpha h_1} Q, \quad M_{24} = -S_2 D_i, \\ M_{33} &= -e^{-2\alpha h_2} Q, \quad M_{34} = -S_3 D_i, \\ M_{44} &= -S_4 D_i, \quad M_{45} = S_4 - S_5 D_i, \\ M_{55} &= S_5 + S_5^T. \end{aligned}$$

Therefore, we finally obtain from (6) and the condition (ii) that

$$\dot{V}(\cdot) + 2\alpha V(\cdot) < x^T(t) J_i x(t), \quad \forall i = 1, 2, \dots, N, \quad t \in R^+.$$

We now apply the condition (i) and Proposition 2.1., the system J_i is strictly complete, and the sets α_i and $\bar{\alpha}_i$ by (2) are well defined such that

$$\begin{aligned} \bigcup_{i=1}^N \alpha_i &= R^n \setminus \{0\}, \\ \bigcup_{i=1}^N \bar{\alpha}_i &= R^n \setminus \{0\}, \quad \bar{\alpha}_i \cap \bar{\alpha}_j = \emptyset, i \neq j. \end{aligned}$$

Therefore, for any $x(t) \in R^n$, $t \in R^+$, there exists $i \in \{1, 2, \dots, N\}$ such that $x(t) \in \bar{\alpha}_i$. By choosing switching rule as $\gamma(x(t)) = i$ whenever $\gamma(x(t)) \in \bar{\alpha}_i$, from (6) we have

$$\dot{V}(\cdot) + 2\alpha V(\cdot) \leq x^T(t) J_i x(t) < 0, \quad t \in R^+,$$

and hence

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \in R^+. \quad (7)$$

Integrating both sides of (7) from 0 to t , we obtain

$$V(t, x_t) \leq V(\phi) e^{-2\alpha t}, \quad \forall t \in R^+.$$

Furthermore, taking condition (3) into account, we have

$$\lambda_1 \|x(t, \phi)\|^2 \leq V(x_t) \leq V(\phi) e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \|\phi\|^2,$$

then

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad t \in R^+,$$

which concludes the proof by the Lyapunov stability theorem [21].

IV. CONCLUSION

This paper has proposed a switching design for the exponential stability of switched linear systems with interval time-varying delays. Based on the improved Lyapunov-Krasovskii functional, a switching rule for the exponential stability for the system is designed via linear matrix inequalities.

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Guaranteed cost control for Hopfield neural networks with interval time-varying delay



Guaranteed cost control for Hopfield neural networks with interval time-varying delay

- ♠ Introduction
- ♠ Main Results
- ♠ Conclusion
- ♠ Acknowledgment
- ♠ References



Introduction

In this paper, we investigate the guaranteed cost control for Hopfield delayed neural networks problem. The novel features here are that the delayed neural network under consideration is with various globally Lipschitz continuous activation functions, and the time-varying delay function is interval, non-differentiable. A nonlinear cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. Based on constructing a set of augmented Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, new delay-dependent criteria for guaranteed cost control via memoryless feedback control is established in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing MATLABs LMI Control Toolbox.

Main Results

Consider the following Hopfield neural networks with interval time-varying delay:

$$\begin{aligned}\dot{x}(t) &= -A_{\gamma(t)}x(t) + W_{0\gamma(t)}f(x(t)) + W_{1\gamma(t)}g(x(t-h(t))) \\ &\quad + B_{\gamma(t)}u(t), \quad t \geq 0, \\ x(t) &= \phi(t), \quad t \in [-h_1, 0],\end{aligned}\tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state of the neural, $u(\cdot) \in L_2([0, t], \mathbb{R}^m)$ is the control; n is the number of neurals, and

$$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T,$$

$$g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t))]^T,$$

are the activation functions;

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$A = \text{diag}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n)$, $\bar{a}_i > 0$ represents the self-feedback term; $B \in R^{n \times m}$ is control input matrix; W_0, W_1 denote the connection weights, the discretely delayed connection weights and the distributively delayed connection weight, respectively; The time-varying delay function $h(t)$ satisfies the condition

$$0 \leq h_0 \leq h(t) \leq h_1,$$

The initial functions $\phi(t) \in C^1([-h_1, 0], R^n)$, with the norm

$$\|\phi\| = \sup_{t \in [-h_1, 0]} \sqrt{\|\phi(t)\|^2 + \|\dot{\phi}(t)\|^2}.$$

In this paper we consider various activation functions and assume that the activation functions $f(\cdot), g(\cdot)$ are Lipschitzian with the Lipschitz constants $f_i, e_i > 0$:

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$$\begin{aligned} |f_i(\xi_1) - f_i(\xi_2)| &\leq f_i |\xi_1 - \xi_2|, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in \mathbb{R}, \\ |g_i(\xi_1) - g_i(\xi_2)| &\leq e_i |\xi_1 - \xi_2|, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in \mathbb{R}, \end{aligned} \quad (2)$$

The performance index associate with the system (1) is the following function

$$J = \int_0^\infty f^0(t, x(t), x(t-h(t)), u(t)) dt, \quad (3)$$

where $f^0(t, x(t), x(t-h(t)), u(t)) : R^+ \times R^n \times R^n \times R^m \rightarrow R^+$, is a nonlinear cost function satisfies

$$\exists Q_1, Q_2, R : f^0(t, x, y, u) \leq Q_1 x, x + Q_2 y, y + R u, u, \quad (4)$$

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for all $(t, x, u) \in R^+ \times R^n \times R^m$ and $Q_1, Q_2 \in R^{n \times n}, R \in R^{m \times m}$, are given symmetric positive definite matrices. The objective of this paper is to design a memoryless state feedback controller $u(t) = Kx(t)$ for system (1) and the cost function (3) such that the resulting closed-loop system

$$\dot{x}(t) = (A + BK)x(t) + W_0 f(x(t)) + W_1 g(x(t - h(t))), \quad (5)$$

is exponentially stable and the closed-loop value of the cost function (3) is minimized.

Definition 1 Given $\alpha > 0$. The zero solution of closed-loop system (5) is α -exponentially stabilizable if there exist a positive number $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\|x(t, \phi)\| \leq Ne^{-\alpha t} \|\phi\|, \quad \forall t \geq 0.$$

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Definition 2 Consider the control system (1). If there exist a memoryless state feedback control law $u^*(t) = Kx(t)$ and a positive number J^* such that the zero solution of the closed-loop system (5) is exponentially stable and the cost function (3) satisfies $J \leq J^*$, then the value J^* is a guaranteed costant and $u^*(t)$ is a guaranteed cost control law of the system and its corresponding cost function. We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 1 (Schur complement lemma [17]). Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

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Proposition 2(Integral matrix inequality [18]). For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds

$$\left(\int_0^\gamma \omega(s) ds \right)^T M \left(\int_0^\gamma \omega(s) ds \right) \leq \gamma \left(\int_0^\gamma \omega^T(s) M \omega(s) ds \right)$$

In this section, we give a design of memoryless guaranteed feedback cost control for neural networks (1). Let us set

$$W_{11} = -[P + \alpha I]A - A^T[P + \alpha I] - 2BB^T + 0.25BRB^T + \sum_{i=0}^1 G_i,$$

$$W_{12} = P + AP + 0.5BB^T,$$

$$W_{13} = e^{-2\alpha h_0} H_0 + 0.5BB^T + AP,$$

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$$W_{14} = 2e^{-2\alpha h_1} H_1 + 0.5BB^T + AP,$$

$$W_{15} = P0.5BB^T + AP,$$

$$W_{22} = \sum_{i=0}^1 W_i D_i W_i^T + \sum_{i=0}^1 h_i^2 H_i + (h_1 - h_0)U - 2P - BB^T,$$

$$W_{23} = P, \quad W_{24} = P, \quad W_{25} = P,$$

$$W_{33} = -e^{-2\alpha h_0} G_0 - e^{-2\alpha h_0} H_0 - e^{-2\alpha h_1} U + \sum_{i=0}^1 W_i D_i W_i^T,$$

$$W_{34} = 0, \quad W_{35} = -2\alpha h_1 U,$$

$$W_{44} = \sum_{i=0}^1 W_i D_i W_i^T - e^{-2\alpha h_1} U - e^{-2\alpha h_1} G_1 - e^{-2\alpha h_1} H_1,$$

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$$W_{45} = e^{-2\alpha h_1} U,$$

$$W_{55} = -e^{-2\alpha h_1} U + W_0 D_0 W_0^T,$$

$$E = \text{diag}\{e_i, i = 1, \dots, n\}, \quad F = \text{diag}\{f_i, i = 1, \dots, n\},$$

$$\lambda_1 = \lambda_{\min}(P^{-1}),$$

$$\begin{aligned} \lambda_2 = & \lambda_{\max}(P^{-1}) + h_0 \lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^1 G_i\right)P^{-1}\right] \\ & + h_1^2 \lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^1 H_i\right)P^{-1}\right] + (h_1 - h_0) \lambda_{\max}(P^{-1} U P^{-1}). \end{aligned}$$

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Theorem 1 Consider control system (1) and the cost function (3). If there exist symmetric positive definite matrices P, U, G_0, G_1, H_0, H_1 , and diagonal positive definite matrices $D_i, i = 0, 1$ satisfying the following LMIs

$$\begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ * & W_{22} & W_{23} & W_{24} & W_{25} \\ * & * & W_{33} & W_{34} & W_{35} \\ * & * & * & W_{44} & W_{45} \\ * & * & * & * & W_{55} \end{bmatrix} < 0, \quad (6)$$

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$$\begin{bmatrix} -PA - A^T P - \sum_{i=0}^1 e^{-2\alpha h_i} H_i & 2PF & PQ_1 \\ * & -D_0 & 0 \\ * & * & -Q_1^{-1} \end{bmatrix} < 0, \quad (7)$$

$$\begin{bmatrix} W_1 D_1 W_1^T - e^{-2\alpha h_1} U & 2PE & PQ_2 \\ * & -D_1 & 0 \\ * & * & -Q_2^{-1} \end{bmatrix} < 0, \quad (8)$$

then

$$u(t) = -\frac{1}{2} B^T P^{-1} x(t), \quad t \geq 0. \quad (9)$$

is a guaranteed cost control and the guaranteed cost value is given by

$$J^* = \lambda_2 \|\phi\|^2.$$

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Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_1}{\lambda_2}} e^{-\alpha t} \|\phi\|, \quad \forall t \geq 0.$$

Proof. Let $Y = P^{-1}$, $y(t) = Yx(t)$. Using the feedback control (5) we consider the following Lyapunov-Krasovskii functional

$$V(t, x_t) = \sum_{i=1}^6 V_i(t, x_t),$$

$$V_1 = x^T(t) Y x(t),$$

$$V_2 = \int_{t-h_0}^t e^{2\alpha(s-t)} x^T(s) Y G_0 Y x(s) ds,$$

$$V_3 = \int_{t-h_1}^t e^{2\alpha(s-t)} x^T(s) Y G_1 Y x(s) ds,$$

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$$V_4 = h_0 \int_{-h_0}^0 \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y H_0 Y \dot{x}(\tau) d\tau ds,$$

$$V_5 = h_1 \int_{-h_1}^0 \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y H_1 Y \dot{x}(\tau) d\tau ds,$$

$$V_6 = (h_1 - h_0) \int_{t-h_1}^{t-h_0} \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y U Y \dot{x}(\tau) d\tau ds.$$

It easy to check that

$$\lambda_1 \|x(t)\|^2 \leq V(t, x_t) \leq \lambda_2 \|x_t\|^2, \quad \forall t \geq 0, \quad (10)$$

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Taking the derivative of $V_i, 1, 2, \dots, 6$, we have

$$\begin{aligned}\dot{V}_1 &= 2x^T(t)Y\dot{x}(t) \\ &= y^T(t)[-PA^T - AP]y(t) - y^T(t)BB^Ty(t) \\ &\quad + 2y^T(t)W_0f(\cdot)y(t) + 2y^T(t)W_1g(\cdot)y(t)\end{aligned}$$

$$\dot{V}_2 = y^T(t)G_0y(t) - e^{-2\alpha h_0}y^T(t-h_0)G_0y(t-h_0) - 2\alpha V_2;$$

$$\dot{V}_3 = y^T(t)G_1y(t) - e^{-2\alpha h_1}y^T(t-h_1)G_1y(t-h_1) - 2\alpha V_3;$$

$$\dot{V}_4 = h_0^2\dot{y}^T(t)H_0\dot{y}(t) - h_1e^{-2\alpha h_0}\int_{t-h_0}^t\dot{x}^T(s)H_0\dot{x}(s)ds - 2\alpha V_4;$$

$$\dot{V}_5 = h_1^2\dot{y}^T(t)H_1\dot{y}(t) - h_1e^{-2\alpha h_1}\int_{t-h_1}^t\dot{y}^T(s)H_1\dot{y}(s)ds - 2\alpha V_5;$$

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$$\begin{aligned}\dot{V}_6 = & (h_1 - h_0)^2 \dot{y}^T(t) U \dot{y}(t) \\ & - (h_1 - h_0) e^{-2\alpha h_1} \int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds - 2\alpha V_6.\end{aligned}$$

we obtain

$$\begin{aligned}\dot{V}(\cdot) + 2\alpha V(\cdot) \leq & \zeta^T(t) \mathcal{E} \zeta(t) + y^T(t) S_1 y(t) \\ & + y^T(t - h(t)) S_2 y(t - h(t)) \\ & - f^0(t, x(t), x(t - h(t)), u(t))\end{aligned}\quad (11)$$

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where

$\zeta(t) = [y(t), \dot{y}(t), y(t-h_0), y(t-h_1), y(t-h(t)), f(\cdot), g(\cdot)]$, and

$$\mathcal{E} = \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ * & W_{22} & W_{23} & W_{24} & W_{25} \\ * & * & W_{33} & W_{34} & W_{35} \\ * & * & * & W_{44} & W_{45} \\ * & * & * & * & W_{55} \end{bmatrix}$$

$$S_1 = -PA - A^T P - \sum_{i=0}^1 e^{-2\alpha h_i} H_i + 4PFD_0^{-1}FP + PQ_1P,$$

$$S_2 = W_1 D_1 W_1^T - e^{-2\alpha h_2} U + 4PED_1^{-1}EP + PQ_2P.$$

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Note that by the Schur complement lemma, Proposition 1, the conditions $S_1 < 0$ and $S_2 < 0$ are equivalent to the conditions (7) and (8), respectively. Therefore, by condition (6), (7), (8), we obtain from (11) that

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \geq 0. \quad (12)$$

Integrating both sides of (12) from 0 to t , we obtain

$$V(t, x_t) \leq V(\phi) e^{-2\alpha t}, \quad \forall t \geq 0.$$

Furthermore, taking condition (10) into account, we have

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$$\lambda_1 \|x(t, \phi)\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \|\phi\|^2,$$

then

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad t \geq 0,$$

which concludes the exponential stability of the closed-loop system (5). To prove the optimal level of the cost function (3), we derive from (11) and (6) - (8) that

$$\dot{V}(t, z_t) \leq -f^0(t, x(t), x(t-h(t)), u(t)), \quad t \geq 0. \quad (13)$$

Integrating both sides of (13) from 0 to t leads to

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$$\int_0^t f^0(t, x(t), x(t-h(t)), u(t)) dt \leq V(0, z_0) - V(t, z_t) \leq V(0, z_0),$$

due to $V(t, z_t) \geq 0$. Hence, letting $t \rightarrow +\infty$, we have

$$J = \int_0^\infty f^0(t, x(t), x(t-h(t)), u(t)) dt \leq V(0, z_0) \leq \lambda_2 \|\phi\|^2 = J^*.$$

This completes the proof of the theorem.

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Example 1 Consider the neural networks with interval time-varying delays (1), where

$$A = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, W_0 = \begin{bmatrix} -0.1 & 0.2 \\ 0.3 & -0.3 \end{bmatrix}, W_1 = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & -0.4 \end{bmatrix}, B = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, F = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, Q_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}, Q_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.5 \end{bmatrix},$$

$$R = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$

$$\begin{cases} h(t) = 0.1 + 1.1 \sin^2 t & \text{if } t \in \mathcal{I} = \bigcup_{k \geq 0} [2k\pi, (2k+1)\pi] \\ h(t) = 0 & \text{if } t \in \mathbb{R}^+ \setminus \mathcal{I}, \end{cases}$$

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Note that $h(t)$ is non-differentiable, therefore, the stability criteria proposed in [5, 6, 7, 12, 15] are not applicable to this system.

Given $\alpha = 0.3$, $h_0 = 0.1$, $h_1 = 1.2$, by using the Matlab LMI toolbox, we can solve for P , U , G_0 , G_1 , H_0 , H_1 , D_0 , and D_1 which satisfy the conditions (3.1)-(3.3) in Theorem 1. A set of solutions are

$$\begin{aligned}
 P &= \begin{bmatrix} 2.4272 & -0.2546 \\ -0.2546 & 1.3172 \end{bmatrix}, & U &= \begin{bmatrix} 7.3269 & -0.1820 \\ -0.1820 & 7.6681 \end{bmatrix}, \\
 G_0 &= \begin{bmatrix} 4.4596 & 0.0397 \\ 0.0397 & 4.2369 \end{bmatrix}, & G_1 &= \begin{bmatrix} 5.2694 & 0.0114 \\ 0.0114 & 5.0125 \end{bmatrix}, \\
 H_0 &= \begin{bmatrix} 4.6455 & 0.0452 \\ 0.0452 & 4.5104 \end{bmatrix}, & H_1 &= \begin{bmatrix} 5.3005 & 0.0233 \\ 0.0233 & 5.2306 \end{bmatrix}, \\
 D_0 &= \begin{bmatrix} 6.0011 & 0 \\ 0 & 6.0011 \end{bmatrix}, & D_1 &= \begin{bmatrix} 5.7809 & 0 \\ 0 & 5.7809 \end{bmatrix}.
 \end{aligned}$$

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Then

$$u(t) = -0.0292x_1(t) - 0.0816x_2(t), t \geq 0$$

is a guaranteed cost control law and the cost given by

$$J^* = 15.4631 \|\phi\|^2.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 0.1614e^{-0.3t} \|\phi\|, \forall t \geq 0.$$

Conclusions

In this paper, the problem of guaranteed cost control for Hopfield neural networks with interval nondifferentiable time-varying delay has been studied. A nonlinear quadratic cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of time-varying Lyapunov-Krasovskii functional combined with Newton-Leibniz formula, a memoryless state feedback guaranteed cost controller design has been presented and sufficient conditions for the existence of the guaranteed cost state-feedback for the system have been derived in terms of LMIs.

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Hanoi, 3 June 2013

Dr. Kreangkri Ratchagit
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Dear Dr. Ratchagit,

It is my pleasure to invite you to visit our Institute from June 19-26, 2013. During the visit you are invited to collaborate with Prof. Vu Ngoc Phat's research group at the Department of Control and Optimization.

Your accommodation and living expenses shall be financed by the Department of Control and Optimization. We hope very much that you will be able to accept our invitation.

I am looking forward to seeing you in Hanoi.

Sincerely yours

Vu N. Phat